8.5 Properties of Logarithmic Functions

$\boxed{\text{old}}$ Exponential Function Rules

Product Rule: $(a^x)(a^y) = a^{x+y}$

Negative Exponent Rule: $a^{-x} = \dfrac{1}{a^x}$

Quotient Rule: $\dfrac{a^x}{a^y} = a^{x-y}$

Power Raised to a Power Rule: $(a^x)^y = a^{xy}$

[Examples] Simplify.

① $(5^3)\cdot(5^x) = 5^{3+x}$

② $\dfrac{(10^{5x})}{(10^3)} = 10^{5x-3}$

③ $(e^{2x})^{4y} = e^{8y}$

④ $e^{-2x} = \dfrac{1}{e^{2x}}$.

go backwards

⑤ $e^{2x} = (e^x)^2$

⑥ $10^{5+2x} = (10^5)(10^{2x})$

⑦ $4^{5-2x} = \dfrac{4^5}{4^{2x}}$

$\boxed{\text{new}}$ Properties of Logarithmic Functions

Product Property: $\log_a x + \log_a y = \log_a (xy)$

Quotient Property: $\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$

Power Property: $\log_a x^n = n \log_a x$

STRATEGIES FOR EXPANDING

1. change radicals to rational (fraction) exponents
2. expand the multiplication (or division)
3. move exponents last (Power Property)

[Examples] Expand.

① $\log_5\left(\frac{\sqrt[6]{x}}{y^2}\right) = \log_5 \frac{x^{\frac{1}{6}}}{y^2} = \log_5 x^{\frac{1}{6}} - \log_5 y^2$

② $\log_3(2x^3) = \log_3 2 + \log_3 x^3 = \log_3 2 + 3\log_3 x.$

③ $\log_{10}(5x^3y) = \log 5 + \log x^3 + \log y = \log 5 + 3\log x + \log y$

④ $\log_7\left(\frac{x^3}{y}\right) = \log_7 x^3 - \log_7 y = 3\log_7 x - \log_7 y$

⑤ $\log_4\left(\frac{5a^2}{b^2}\right) = \log_4 5a^2 - \log_4 b^2 = \log_4 5 + \log_4 a^2 - \log_4 b^2 = \log_4 + 2\log_4 a - 2\log_4 b.$

⑥ $\ln\left(\frac{3\sqrt[3]{x}}{yz}\right) = \ln\left(\frac{3x^{1/3}}{yz}\right) = \ln 3x^{1/3} - \ln yz = \ln 3 + \frac{1}{3}\ln x - \ln yz.$

STRATEGIES FOR CONDENSING

1. If there is a number in front (coefficient), move it to the back to an exponent (Power Prop.)
2. Write exponents as radicals.
3. If number is raised to power, simplify
4. Condense add back to multiplication & subtraction back to divison
5. Final answer should be only 1 log, and No rational exponents.

[Examples] Condense.

① $\log_2 20 + 2\log_2 2 + \frac{1}{3}\log_2 x$ $= \log_2 20 + \log_2 2^2 + \log_2 x^{1/3}$

$= \log_2 20 + \log_2 4 + \log_2 \sqrt[3]{x}$

$= \log_2(20 \cdot 4 \cdot \sqrt[3]{x})$

$= \log_2(80\sqrt[3]{x}).$