8.2 Logarithmic Functions

$\boxed{\text{old-A}}$ Solving Equations

① $-7x-3x+2=-8x-8$

$-10x+2=-8x-8$

$-2x+2=-8$

$-2x=-10$

$x=5$

② $5=\sqrt{r-3}+2$

$3=\sqrt{r-3}$

$(3)^2=(\sqrt{r-3})^2$

$9=r-3$

$12=r$.

③ $x^2=9$

$\sqrt{x^2}=\sqrt{9}$

$x=\pm 3$

using <u>inverse operations</u> to solve equations

$\boxed{\text{old}-\text{B}}$ Inverse Functions

Let's recall: the relationship between 2 inverse functions

<u>graphically</u>

- reflection across $y=x$ line (passes horizontal line test)

<u>numerically</u>

- x & y interchange roles

<u>algebraically</u>

$f(f^{-1}(x))=x$.

(Example) Find the inverse function.

$f(x)=2x-3$

$y=2x-3$

$x=2y-3$

$x+3=2y$

$\frac{x+3}{2}=y$

$f^{-1}(x)=\frac{x+3}{2}$

$\boxed{\text{new}}$ Logarithmic Functions

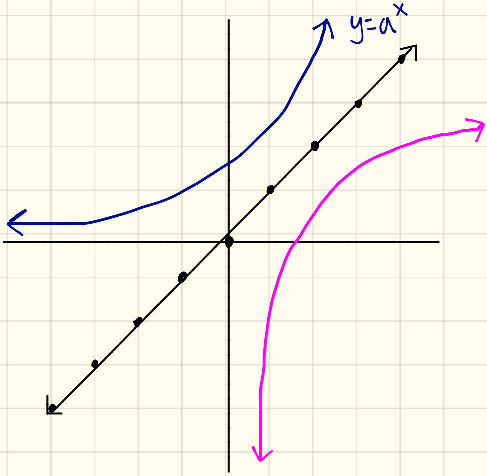
Let's consider some exponential functions. Find x.

① $2^x = 8$ ② $3^x = 9$ ③ $5^x = 120$ Dilemma:

$x = 3$ $x = 2$ $x = ?$ How do we solve exponential functions?

To solve this dilemma, we need to find exponential function's inverse!

Let's consider $y = a^x$. Graph the arbitrary function.



Now graph the arbitrary inverse function by reflecting across $y = x$ line.

The model for this new inverse equation is
$\log_a y = x$.

LOGARITHMIC FUNCTIONS

The inverse of an exponential function is a log function.

$$a^x = y \Longleftrightarrow \log_a y = x.$$

We need to convert exponential equations into log equations to solve exponential functions.

"All you need is BOB" to help you convert back and forth from exponentials to logs & vice versa.

Base

Opposite

Back

$a^x = y \iff \log_a y = x$

[Examples] Rewrite into the inverse form.

① $3^y = 5$

$\log_3 5 = y$

② $5^x = 125$

$\log_5 125 = x$

③ $z^0 = e$

$\log_z e = 0.$

④ $\log_{10} y = 3$

$10^3 = y$

⑤ $\log_2 x = 4$

$2^4 = x$

⑥ $\log_z 0 = e$

$z^e = 0$

Let's figure out some facts!

$a^1 = a$: $\log_a a = 1$, $a^{\log a} = 1$ $\Longrightarrow$ Makes sense because they are inverses! (Right?)

[Examples] Rewrite into the inverse form. 2 ways to do now! either know the conversion or take the log of the exponentials base.

① $3^y = 5$

method 1: know conversion

$\log_3 5 = y$

method 2: take log on both sides

$3^y = 5$

$\log_3 3^y = \log_3 5$

$y = \log_3 5$

[Examples] Solving exponentials & logarithmic eqtns.

① $2^x = 8$

$\log_2 8 = x$

$3 = x$

② $5^x = 25$

$\log_5 25 = x$

$2 = x$

What about $5^x = 120$?

$\log_5 120 = x$

we still need a way to compute logs!

CHANGE OF BASE FORMULA:

Let's consider $\log y = x$. When log functions doesn't seem to have a base, the base is 10!

$\log_a y = x$

$x = \frac{\log y}{\log a}$

* insert value in calculator.

$5^x = 120$

$\log_5 120 = x$

$x = \frac{\log 120}{\log 5} \approx 2.97$