8.1 Inverse Functions

$\boxed{\text{Old}}$ Rearranging Formulas – Solving for variables

① $n + c = m$ (solve for n)

$n = m - c$

② $j = \dfrac{n}{p}$ (solve for p)

$jp = n$

$p = \dfrac{n}{j}$

③ $\dfrac{p}{q} - m = n$ (solve for p)

$\dfrac{p}{q} = n + m$

$p = q(n+m)$

④ $2y + a = x$ (solve for y)

$2y = x - a$

$y = \dfrac{x-a}{2}$

⑤ $P = 2l + 2w$ (solve for w)

$P - 2l = 2w$

$\dfrac{P-2l}{2} = w$

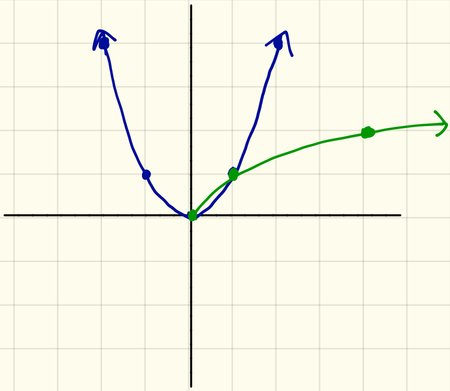
⑥ $ch + es = ni$ (solve for e)

$es = ni - ch$

$e = \dfrac{ni - ch}{s}$

$\boxed{\text{new}}$ Inverse Functions

Let's consider the 2 functions: $f(x) = x^2$ and $g(x) = \sqrt{x}$. Graph, create a T-chart and analyze.



x	$y=x^2$
0	0
1	1
2	4
3	9
4	16
⋮	⋮

x	$y=\sqrt{x}$
0	0
1	1
4	2
9	3
16	4
⋮	⋮

- What is the relationship between $y=x^2$ and $y=\sqrt{x}$?

The 2 functions are inverse functions

- What can you notice about the inverse relationship on the T-chart?

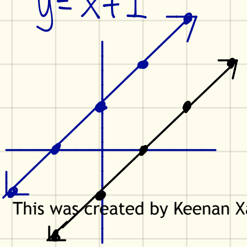
The x and y values switched places.

What do you notice about the inverse relationship on the Cartesian Plane?

The 2 functions are a reflection about the line $y=x$.

Let's look at some other cases:

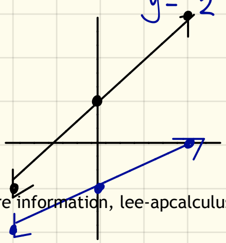
What is the inverse of $y = x-1$?



x	$y=x+1$
0	1
1	2
5	6
7	8
⋮	⋮

x	$y=x-1$
1	0
2	1
6	5
8	7
⋮	⋮

What is the inverse of $y = 2x+1$?

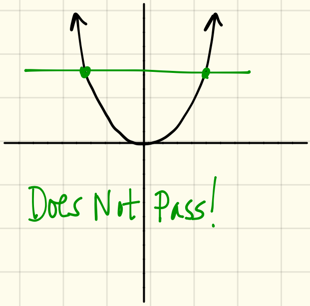


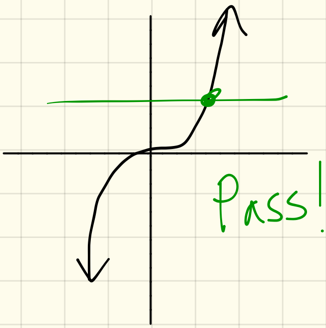
x	$y=\frac{x}{2}-1$
0	-1
2	0
-2	-2

x	$y=x-1$
-1	0
0	2
-2	-2

• Inverse function notation. $f(x)$ = function
$f^{-1}(x)$ = inverse function of $f(x)$.

• To determine graphically if a function will have an inverse, use the Horizontal Line Test.





Finding the inverse of a Function Algebraically:
step 1 – In the equation, replace $f(x)$ by y. (if necessary)
step 2 – interchange the roles of x & y, and solve for y.
step 3 – Replace y by $f^{-1}(x)$ in the "new" inverse equation.

[Examples] Find inverse algebraically.

① $f(x) = 2x-5$

$y = 2x-5$

$x = 2y-5$

$x+5 = 2y$

$\frac{x+5}{2} = y$

$f^{-1}(x) = \frac{x+5}{2}$

② $f(x) = 3x+7$

$y = 3x+7$

$x = 3y+7$

$x-7 = 3y$

$\frac{x-7}{3} = y$

$f^{-1}(x) = \frac{x-7}{3}$

③ $f(x) = \frac{x+5}{7}$

$y = \frac{x+5}{7}$

$x = \frac{y+5}{7}$

$7x = y+5$

$7x-5 = y$

$f^{-1}(x) = 7x-5$

④ $y = \dfrac{x-1}{5}$

$x = \dfrac{y-1}{5}$

$5x = y-1$

$5x+1 = y$

$f^{-1}(x) = 5x+1.$

⑤ $y = \sqrt{x+7}$

$x = \sqrt{y+7}$

$(x)^2 = (\sqrt{y+7})^2$

$x^2 = y+7$

$x^2+7 = y$

$f^{-1}(x) = x^2+7$

⑥ $g(x) = \sqrt[3]{5-x}$

$y = \sqrt[3]{5-x}$

$(y)^3 = (\sqrt[3]{5+x})^3$

$y^3 = 5+x$

$y^3-5 = x$

$x^3-5 = y$

$g^{-1}(x) = x^3-5$

⑦ $f(x) = \dfrac{x-1}{x+5}$

$y = \dfrac{x-1}{x+5}$

$x = \dfrac{y-1}{y+5}$

$x(y+5) = y-1$

$xy+5x = y-1$

$xy-y = -1-5x$

$y(x-1) = -1-5x$

$y = \dfrac{-1-5x}{x-1}$

$f^{-1}(x) = \dfrac{-1-5x}{x-1}$

⑧ $f(x) = \dfrac{x+3}{x-1}$

$y = \dfrac{x+3}{x-1}$

$x = \dfrac{y+3}{y-1}$

$x(y-1) = y+3$

$xy-x = y+3$

$xy-x = y+3$

$xy-y = x+3$

$y(x-1) = x+3$

$y = \dfrac{x+3}{x-1}.$