

7.6 Solving Radical Functions

Old Solving Quadratics Equations

$$\begin{aligned} \textcircled{1} \quad (a+1)^2 &= 4 \\ \sqrt{(a+1)^2} &= \sqrt{4} \\ a+1 &= \pm 2 \\ \begin{array}{r} -1 \quad -1 \\ \hline a = -1 \pm 2 \end{array} \end{aligned}$$

$$a = -1 + 2 \text{ or } a = -1 - 2 \\ = 1 \text{ or } a = -3$$

$$\begin{aligned} \textcircled{2} \quad (m-6)^2 &= 10 \\ \sqrt{(m-6)^2} &= \sqrt{10} \\ m-6 &= \pm\sqrt{10} \\ +6 &= +6 \\ \hline m &= 6 \pm \sqrt{10} \end{aligned}$$

$$m = 6 + \sqrt{10} \text{ or } m = 6 - \sqrt{10}$$

$$\begin{aligned} \textcircled{3} \quad (x-9)^2 &= -4 \\ \sqrt{(x-9)^2} &= \sqrt{-4} \\ x-9 &= \pm 2i \\ +9 &= +9 \\ \hline x &= 9 \pm 2i \end{aligned}$$

$$x = 9 + 2i \text{ or } 9 - 2i$$

New Solving Radical Equations

Let's recall the inverse operations for each operation:

Addition inverses Subtraction

Multiplication inverses Division

Square Root inverses Squaring

n^{th} root inverses raise to the n^{th} power.

Steps to Solving Radical Equations:

1. Isolate the radical term.
2. Raise both sides of the equation to the power to the root of the radical.
3. Simplify & solve.

[Examples] Solve.

$$\begin{aligned} \textcircled{1} \quad 5 + \sqrt{x+1} &= 16 \\ \sqrt{x+1} &= 11 \\ (\sqrt{x+1})^2 &= (11)^2 \\ x+1 &= 121 \\ x &= 120. \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad 7\sqrt{5x-7} &= 84 \\ \frac{7\sqrt{5x-7}}{7} &= \frac{84}{7} \\ \sqrt{5x-7} &= 12 \\ (\sqrt{5x-7})^2 &= (12)^2 \\ 5x-7 &= 144 \\ 5x &= 151 \\ x &= \frac{151}{5} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad 7\sqrt[3]{5x-7} &= 84 \\ \frac{7\sqrt[3]{5x-7}}{7} &= \frac{84}{7} \\ \sqrt[3]{5x-7} &= 12 \\ (\sqrt[3]{5x-7})^3 &= (12)^3 \\ 5x-7 &= 1728 \\ 5x &= 1735 \\ x &= 347. \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad \sqrt{7x+2} &= \sqrt{3x-2} \\ (\sqrt{7x+2})^2 &= (\sqrt{3x-2})^2 \\ 7x+2 &= 3x-2 \\ 10x &= -4 \\ x &= \frac{-4}{10} = -\frac{2}{5} \end{aligned}$$

$$\begin{aligned} \textcircled{5} \quad \sqrt{x+6} &= 2\sqrt{x-1} \\ (\sqrt{x+6})^2 &= (2\sqrt{x-1})^2 \\ x+6 &= 4(x-1) \\ x+6 &= 4x-4 \\ -3x+6 &= -4 \\ -3x &= -10 \\ x &= -\frac{10}{3}. \end{aligned}$$

$$\begin{aligned} \textcircled{6} \quad \sqrt{-3x+33} &= 5-x \\ (\sqrt{-3x+33})^2 &= (5-x)^2 \\ -3x+33 &= 25-10x+x^2 \\ 0 &= x^2-7x-8 \\ 0 &= (x-8)(x+1) \\ x &= 8, \quad x = -1. \end{aligned}$$

$$\textcircled{7} \quad \sqrt{x+3} = -4$$

NO SOLUTION! A square root can NEVER equal a negative number!