6.2 Binomial Expansion

Old: Polynomial Operations – Add, Subtract, Multiply

1. $(4y^3 - 5y^2) + (12y^5 - 2y^3 + 14y^2) = 12y^5 + 2y^3 + 9y^2$
2. $(3y^5 + 8y^3 - 10y^2) - (-12y^5 + 4y^3 + 14y^2) = 15y^5 + 4y^3 - 24y^2$
3. $(-7n^2 + 8n - 4) - (-11n + 2 + 14n^2) = 7n^2 + 19n - 6$
4. $(-10k^2 + 7k + 6k^4) + (-14 - 4k^4 - 14k) = 2k^4 - 10k^2 - 7k - 14$
5. $2x(-2x-3) = -4x^2 - 6x$
6. $(3m-1)(8m+7) = 24m^2 + 13m - 7$
7. $(2a-1)(8a-5) = 16a^2 - 18a + 5$
8. $(4n+1)(2n+6) = 8n^2 + 26n + 6$

New: Binomial Expansion

Let's consider $(x+5)^2$. Expand. $(x+y)(x+y) = x^2 + 2xy + y^2$

Now, let's consider $(x+5)^3$. Expand. $(x+y)(x+y)(x+y) = x^3 + 3x^2y + 3xy^2 + y^3$

What about $(x+y)^5$?

Dilemma: $(x+y)^5$ is ALOT of work to expand!

Is there a more efficient way to expand binomials?

Pascal's Triangle (Binomial Theorem)

The first & last numbers in each row are 1. Beginning with the second row, every other number is by adding the 2 numbers immediately above the number.

$$
\begin{array}{ccccccccccccc|l}
 & & & & & & 1 & & & & & & & (x+y)^0 \\
 & & & & & 1 & & 1 & & & & & & (x+y)^1 \\
 & & & & 1 & & 2 & & 1 & & & & & (x+y)^2 \\
 & & & 1 & & 3 & & 3 & & 1 & & & & (x+y)^3 \\
 & & 1 & & 4 & & 6 & & 4 & & 1 & & & (x+y)^4 \\
 & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & & (x+y)^5 \\
1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1 & \\
 & & & & & & \vdots & & & & & & & \ddots\ (x+y)^n
\end{array}
$$

please note: The exponents in the expansion add to the exponent in the binomial exponent.

$$(x+y)^2 = x^2y^0 + 2x^1y^1 + x^0y^2 \\ = x^2 + 2xy + y^2$$

$$(x+y)^3 = x^3y^0 + 3x^2y^1 + 3xy^2 + x^0y^3 \\ = x^3 + 3x^2y + 3xy^2 + y^3$$

[Example 1] Expand $(x+y)^4$.

$$(x+y)^4 = 1x^4y^0 + 4x^3y^1 + 6x^2y^2 + 4xy^3 + 1x^0y^4 \\ = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

[Example 2] Expand $(x+y)^5$.

$$(x+y)^5 = 1x^5y^0 + 5x^4y^1 + 10x^3y^2 + 10x^2y^3 + 5xy^4 + 1x^0y^5 \\ = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$

[Example 3] Expand $(x+3)^2$

$$(x+3)^2 = 1x^2(3)^0 + 2x(3) + 1x^0(3)^2 \\ = x^2 + 6x + 9$$

[Example 4] Expand $(x+5)^4$

$$(x+5)^4 = 1x^4(5)^0 + 4x^3(5)^1 + 6x^2(5)^2 + 4x(5)^3 + 1x^0(5)^4. \\ = x^4 + 20x^3 + 150x^2 + 500x + 625$$

[Example 5] Expand $(2x+1)^3$

$$(2x+1)^3 = 1(2x)^3(1)^0 + 3(2x)^2(1) + 3(2x)(1)^2 + 1(2x)^0(1)^3 \\ = 8x^3 + 12x^2 + 6x + 1$$

What if the binomial has a <u>negative</u> instead of a positive sign?

Let's consider $(x-y)^3$.

$$
\begin{array}{ccccccccccccccl}
 & & & & & & 1 & & & & & & & & (x-y)^0 \\
 & & & & & 1 & & -1 & & & & & & & (x-y)^1 \\
 & & & & 1 & & -2 & & 1 & & & & & & (x-y)^2 \\
 & & & 1 & & -3 & & 3 & & -1 & & & & & (x-y)^3 \\
 & & 1 & & -4 & & 6 & & -4 & & 1 & & & & (x-y)^4 \\
 & 1 & & -5 & & 10 & & -10 & & 5 & & -1 & & & (x-y)^5 \\
1 & & -6 & & 15 & & -20 & & 15 & & -6 & & 1 & & \\
 & & & & & & \vdots & & & & & & & & \therefore (x-y)^n
\end{array}
$$

every other number gets a negative.

$$
\begin{aligned}
(x-y)^3 &= 1x^3y^0 - 3x^2y^1 + 3xy^2 - 1x^0y^3 \\
&= x^3 - 3x^2y + 3xy^2 - y^3.
\end{aligned}
$$