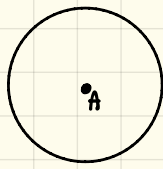


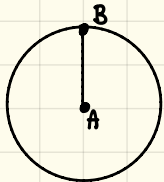
4.3 Writing Equations of Circles, Graph Circles & Change Forms

Old Circle Vocabulary

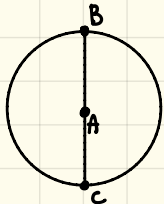
Let's consider a circle in space with a center A. Discuss the relationship between a circle & center of a circle.



Center of the circle is at an equal distance from all points on the circle



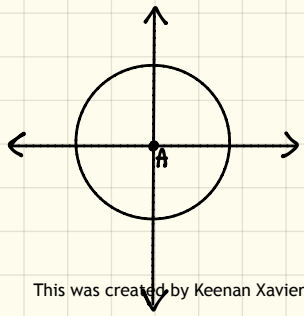
Radius of the circle is the distance from the center to any point on the circle.
(Half the diameter)



Diameter of the circle is the line segment connecting 2 points on the circle & passing through the center of a circle.
(2 times the radius)

New-A Graphing Circles

Let's consider a circle on a coordinate plane.



Standard Form of a Circle
 $(x-h)^2 + (y-k)^2 = r^2$

Center of a circle: (h, k)
Radius of a circle: r

[Example 1] Write an equation of a circle with center $(0,0)$ & radius 3. Sketch the circle.

Standard Form of a Circle

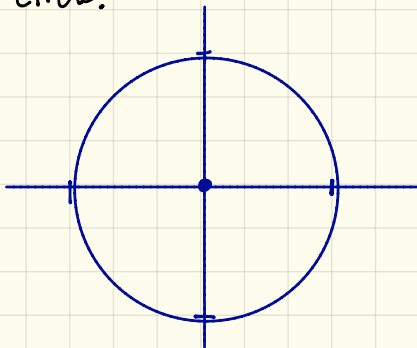
$$(x-h)^2 + (y-h)^2 = r^2$$

Center: $(0,0)$, radius = 3

$$= (x-0)^2 + (y-0)^2 = (3)^2$$

$$= x^2 + y^2 = (3)^2$$

$$= x^2 + y^2 = 9$$



[Example 2] Write an equation of a circle with center $(3,-2)$ and radius of 4. Sketch the circle.

Standard Form of a Circle

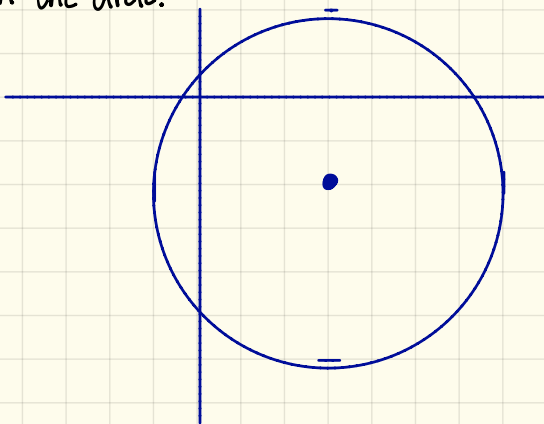
$$(x-h)^2 + (y-h)^2 = r^2$$

Center: $(3,-2)$, radius = 4

$$= (x-3)^2 + (y-(-2))^2 = (4)^2$$

$$= (x-3)^2 + (y+2)^2 = (4)^2$$

$$= (x-3)^2 + (y+2)^2 = 16$$



[Example 3] Write an equation of a circle with center $(-4,0)$ and a diameter of 10.

Standard Form of a Circle

$$(x-h)^2 + (y-h)^2 = r^2$$

Center $(-4,0)$, radius = 5

$$= (x-(-4))^2 + (y-0)^2 = (5)^2$$

$$= (x+4)^2 + y^2 = (5)^2$$

$$= (x+4)^2 + y^2 = 25$$

[Example 4] Identify the center & radius of the circle from the standard form of a circle: $(x-2)^2 + (y+9)^2 = 49$.

Standard Form of a Circle

$$(x-h)^2 + (y-k)^2 = r^2$$

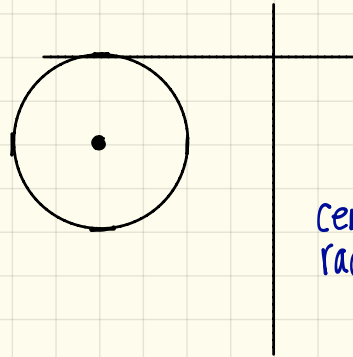
Center: (h, k)

$$(x-2)^2 + (y+9)^2 = 49$$

$$(x-2)^2 + (y-(-9))^2 = (7)^2$$

Center: $(2, 9)$, radius = 7

[Example 6] Find the standard form of a circle using the graph below.



center: $(-4, -2)$
radius = 2

Standard Form of a Circle

$$(x-h)^2 + (y-k)^2 = r^2$$

Center: (h, k)

$$\begin{aligned} &= (x-(-4))^2 + (y-(-2))^2 = (2)^2 \\ &= (x+4)^2 + (y+2)^2 = (2)^2 \\ &= (x+4)^2 + (y+2)^2 = 4. \end{aligned}$$

[New-B] Writing Equations of Circles

Let's recall standard form of a circle: $(x-h)^2 + (y-k)^2 = r^2$ where (h, k) is the center & r is the radius.

General Form of a circle: $ax^2 + by^2 + cx + dy + e = 0$.

Let's consider the an equation general form of a circle:
 $x^2 + y^2 - 4x - 6y - 3 = 0$. Find the center, radius & sketch the graph.

Dilemma: We need to convert the equation from general form to standard form to identify center & radius.

In order to convert from GENERAL to STANDARD, you need to complete the square!

- 1st Group the x terms together & group the y terms together.
- 2nd Move the "c" term to the other side of the equals sign
- 3rd Complete the Square for x & Complete the square for y.
- 4th Factor the left & simplify the right.

$$x^2 + y^2 + 4x + 6y - 3 = 0$$

$$x^2 + 4x + y^2 + 6y - 3 = 0$$

$$x^2 + 4x + \underline{\quad} + y^2 + 6y + \underline{\quad} = 3 + \underline{\quad} + \underline{\quad}$$

$$(x^2 + 4x + (\frac{4}{2})^2) + (y^2 + 6y + (\frac{6}{2})^2) = 3 + (\frac{4}{2})^2 + (\frac{6}{2})^2$$

$$(x^2 + 4x + 4) + (y^2 + 6y + 9) = 3 + 4 + 9$$

$$(x+2)(x+2) + (y+3)(y+3) = 16$$

$$(x+2)^2 + (y+3)^2 = (4)^2$$

Center: $(-2, -3)$, radius = 4.

[Example 1] Sketch the graph of $x^2 + y^2 - 8x + 7 = 0$.

$$x^2 + y^2 - 8x + 7 = 0$$

$$x^2 - 8x + y^2 = -7$$

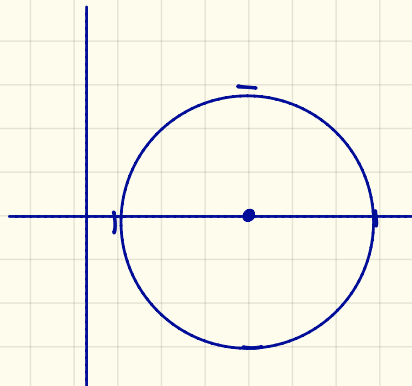
$$(x^2 - 8x + \underline{\quad}) + (y^2) = -7 + \underline{\quad}$$

$$(x^2 - 8x + (\frac{8}{2})^2) + (y^2) = -7 + (\frac{8}{2})^2$$

$$(x^2 - 8x + 16) + y^2 = -7 + 16$$

$$(x-4)(x-4) + y^2 = 9$$

$$(x-4)^2 + y^2 = (3)^2$$



Center: $(4, 0)$, radius = 3

[Example 2] State the center & radius of $2x^2 + 2y^2 - 16x + 4y + 20 = 0$

$$\begin{aligned}2x^2 + 2y^2 - 16x + 4y + 20 &= 0 \\ \div 2 (2x^2 + 2y^2 - 16x + 4y + 20 = 0) \div 2 \\ x^2 + y^2 - 8x + 2y + 10 &= 0 \\ x^2 - 8x + y^2 + 2y &= -10 \\ x^2 - 8x + \underline{\quad} + y^2 + 2y + \underline{\quad} &= -10 + \underline{\quad} + \underline{\quad} \\ (x^2 - 8x + (\frac{8}{2})^2) + (y^2 + 2y + (\frac{2}{2})^2) &= -10 + (\frac{8}{2})^2 + (\frac{2}{2})^2 \\ (x^2 - 8x + 16) + (y^2 + 2y + 1) &= -10 + 16 + 1 \\ (x-4)(x-4) + (y+1)(y+1) &= 7 \\ (x-4)^2 + (y+1)^2 &= (\sqrt{7})^2\end{aligned}$$

Center: $(4, -1)$, radius: $\sqrt{7}$

[Example 3] Write the general form of $(x-4)^2 + (y+3)^2 = 36$

$$\begin{aligned}(x-4)^2 + (y+3)^2 &= 36 \\ (x-4)(x-4) + (y+3)(y+3) &= 36 \\ x^2 - 8x + 16 + y^2 + 6y + 9 - 36 &= 0 \\ x^2 + y^2 - 8x + 6y + 16 + 9 - 36 &= 0 \\ x^2 + y^2 - 8x + 6y - 11 &= 0.\end{aligned}$$