4.1 Graph Parabolas

Old Vertex Form of Quadratics

Let's recall the vertex form for Quadratics:

$$y = a(x-h)^2 + k$$

Vertex : (h, k)

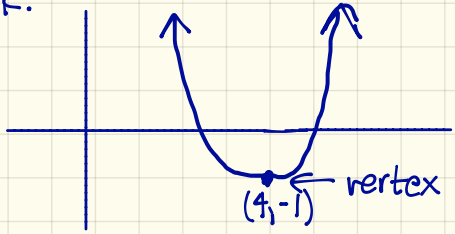
When a is positive, ∪ When a is negative, ∩ (reflection across x-axis)

When $a > 1$, stretch When $0 < a < 1$, shrink

[Examples] Sketch a graph.

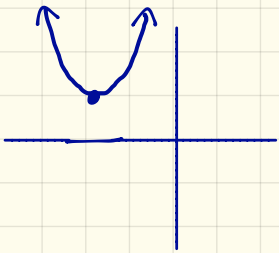
① $y = \frac{1}{2}(x-4)^2 - 1$ shrink!

vertex = $(4, -1)$



② $y = (x+2)^2 + 1$

Vertex: $(-2, 1)$



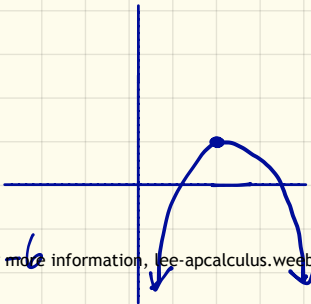
③ $y = x^2 + 8x + 10$

$a = 1$
$b = 8$
$c = 10$

$x = \frac{-b}{2a} = \frac{-8}{2(1)} = -4$

$y = f(-4) = (-4)^2 + 8(-4) + 10 = -6$

Vertex: $(-4, -6)$ $y = 1(x+4)^2 - 6$



new Parabolas

What are parabolas in a coordinate plane?
A parabola is a set of points that is the same distance away from a single point called the focus & a line called a directrix.

A parabola opens up in 4 ways?

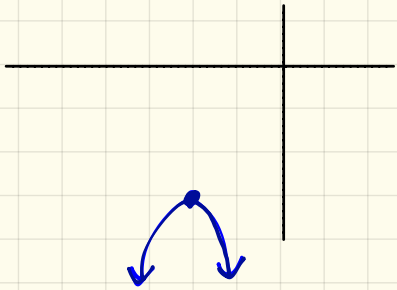
a is pos.  a is neg.  (opens up or down) $y = a(x-h)^2 + k$
vertex: (h, k)

a is pos.  a is neg.  (opens right or left) $x = a(y-k)^2 + h$
vertex: (h, k)

[Examples] Sketch the graph.

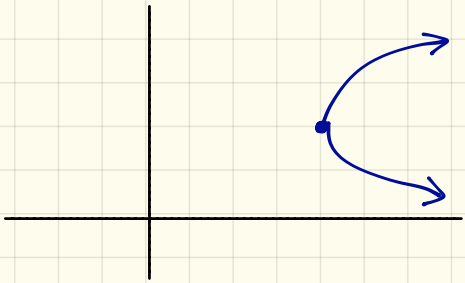
① $y = -(x+2)^2 - 3$

vertex: $(-2, -3)$
no stretch or shrink
↕ (reflection across x-axis

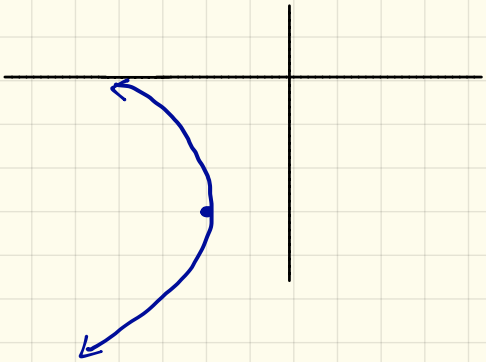


② $x = (y-2)^2 + 4$

vertex: $(4, 2)$
no stretch or shrink
↻



3. $x = -\frac{1}{8}(y+3)^2 - 2$



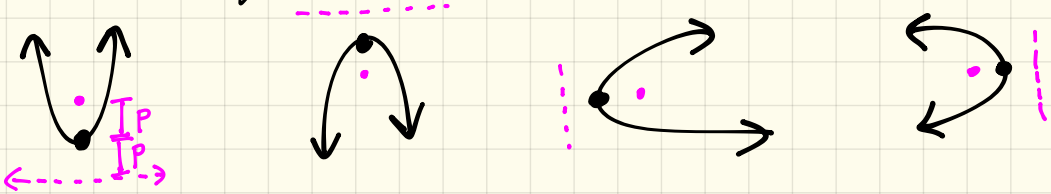
Vertex: $(-2,-3)$
shrink by a factor of $\frac{1}{8}$

Focus & Directrix

A parabola always contains a focus & directrix.

- Focus – point inside the parabola
- Directrix – Line outside the parabola

(Both are equal distance away from the parabola)



Let p be the equal distance the focus & directrix is from the vertex of a parabola.

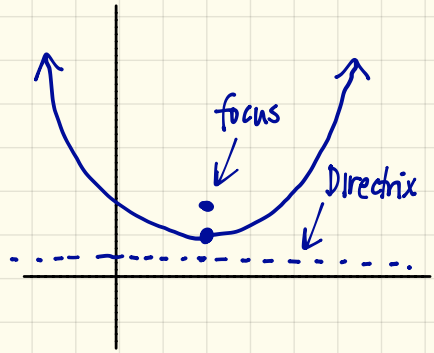
FORMULA for p:

$p = \frac{1}{4a}$

Vertex form: $y = a(x-h)^2 + k$
$x = a(y-k) + h$

Let's consider $y = 3(x-2)^2 + 1$. Sketch the graph. Find the focus & directrix.

- Vertex: (2, 1)
- stretch by a factor of 3
- ↶



Focus is inside of parabola $(h, k+p)$
Directrix is outside of parabola: $y = k-p$

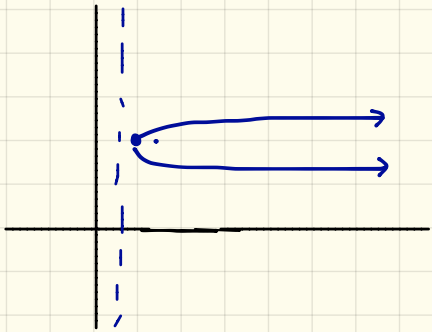
Formula for p: $p = \frac{1}{4a}$

$a = 3$ $\quad p = \frac{1}{4(a)} = \frac{1}{12}$

Focus: $(2, 1+\frac{1}{12}) = (2, \frac{13}{12}) = (2, 1.083)$
Directrix: $y = 1 - \frac{1}{12} = \frac{11}{12} = .916$

Let's consider $x = 3(y-2)^2 + 1$. Sketch the graph. Find the focus & directrix.

- Vertex: (1, 2)
- stretch by a factor of 3
- ↷



Focus is inside of parabola $(k+p, h)$
Directrix is outside of parabola $y = k-p$

Formula for p: $p = \frac{1}{4a}$

$a = 3$ $\quad p = \frac{1}{4(a)} = \frac{1}{12}$

Focus: $(1+\frac{1}{12}, 1) = (\frac{13}{12}, 1) = (1.08\overline{3}, 1)$
Directrix: $x = 1 - \frac{1}{12} = \frac{11}{12} = .91\overline{6}$

Note: It's always good to sketch a picture to help determine focus & directrix.

THINGS TO MEMORIZE:

- Formula for p: $\frac{1}{4a}$
- vertex form for Parabolas

$y = a(x-h)^2 + k$
vertex: (h, k)

Focus: $(h, K+p)$
Directrix: $y = k-p$

$x = a(y-k)^2 + h$
vertex: (k, h)

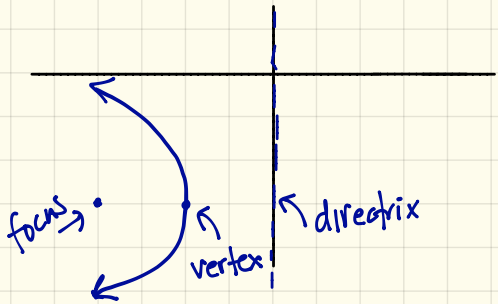
Focus: $(k+p, h)$
Directrix: $y = k-p$

[Examples] Find the focus & directrix for $x = \frac{-1}{8}(y-4)^2 - 2$

① Vertex: $(-2, 4)$
Shrink by a factor of $\frac{1}{8}$

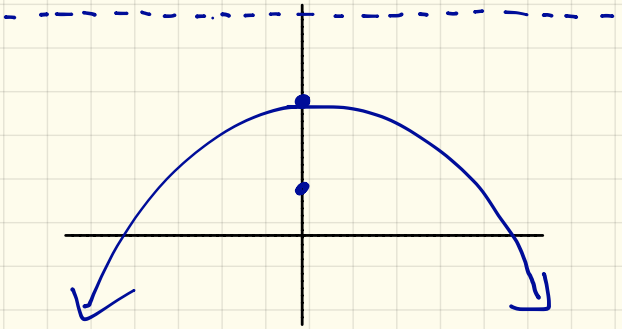
$a = \frac{-1}{8}$

$p = \frac{1}{4(-\frac{1}{8})} = -2$



Focus: $(-2+(-2), -2) = (-4, 2)$
Directrix: $x = -2 - -2 = 0$

② $y = \frac{-1}{8}x^2 + 3$



Vertex: $(0, 3)$
shrink by a factor of $\frac{1}{8}$
↳ reflection across x-axis

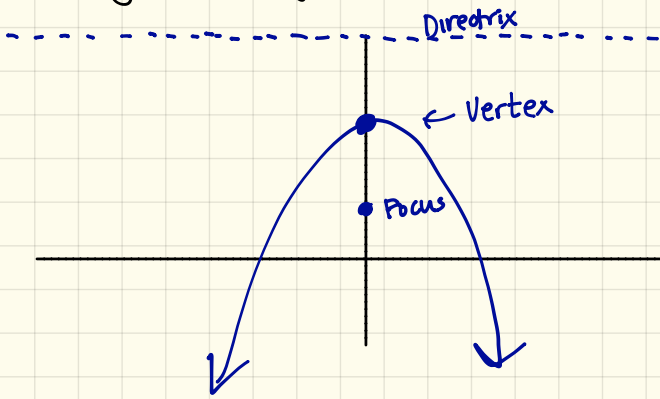
$a = -\frac{1}{8}$

$p = \frac{1}{4(-\frac{1}{8})} = -2$

Focus: $(0, 3+-2) = (0, 1)$
Directrix: $y = 3 - -2 = 5$

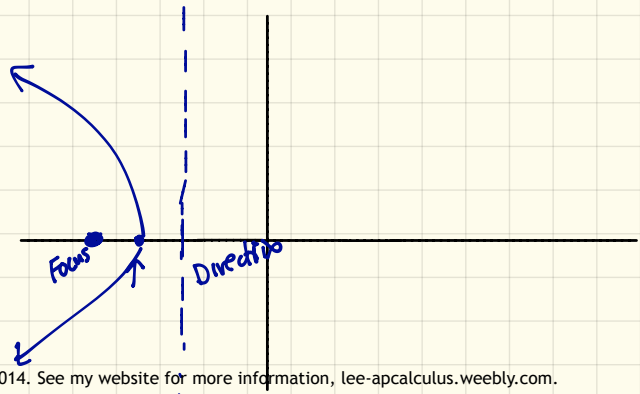
[Examples] Find the vertex when given the focus & directrix.

① Focus: $(0, 1)$
Directrix: $y = 5$



vertex: $(0, 3)$

② Focus: $(-4, 0)$
Directrix: $x = -2$



Vertex: $(-3, 0)$