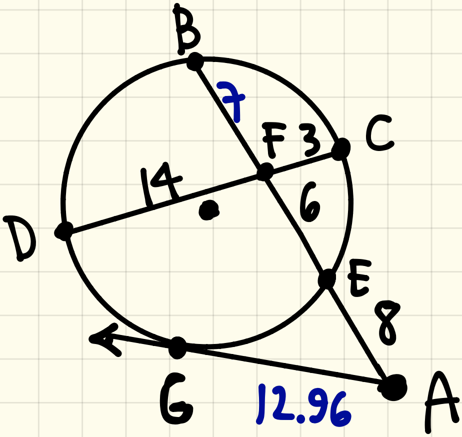
3.4 Chord Properties, Tangent Problems

$\boxed{\text{old}}$ Finding Segment Lengths in Circles

Find $\overline{BF}$ & $\overline{AG}$.



Find $\overline{BF}$.

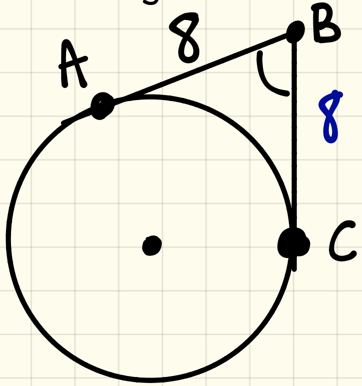
$$\text{part} * \text{part} = \text{part} * \text{part}$$
$$\overline{CF} * \overline{DF} = \overline{BF} * \overline{EF}$$
$$3 * 14 = \overline{BF} * 6$$
$$42 = 6\overline{BF}$$
$$7 = \overline{BF}$$

Find $\overline{AE}$

$$\text{outside} * \text{whole} = \text{outside} * \text{whole}$$
$$\overline{AG} * \overline{AG} = \overline{AE} * \overline{AB}$$
$$\overline{AG}^2 = 8 * 21$$
$$\overline{AG}^2 = 168$$
$$\sqrt{\overline{AG}^2} = \sqrt{168}$$
$$\overline{AG} = \sqrt{168} = 2\sqrt{42} \approx 12.96$$

$\boxed{\text{new}}$ Chord Properties

Let's consider the below diagram. Find $\overline{BC}$.



outside * whole = outside * whole

$\overline{AB} * \overline{AB} = \overline{BC} * \overline{BC}$

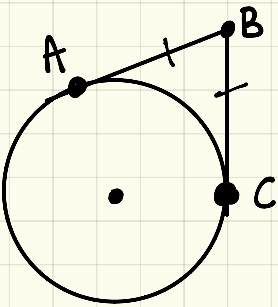
$8 * 8 = \overline{BC} * \overline{BC}$

$64 = \overline{BC}^2$

$\sqrt{64} = \sqrt{\overline{BC}^2}$

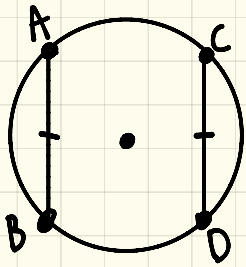
$8 = \overline{BC}.$

1. If 2 segments from the same exterior vertex are tangent to a circle, then they are congruent.



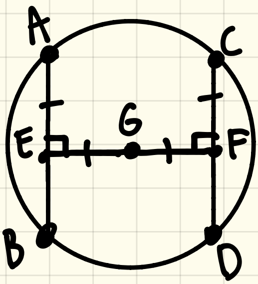
$\overline{AB} \cong \overline{BC}$

1. If 2 chords are congruent, then their corresponding arcs are congruent.



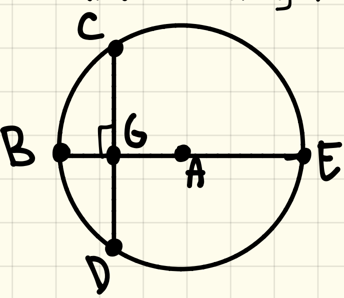
$\overline{AB} \cong \overline{CD}$ and $\overset{\frown}{AB} = \overset{\frown}{CD}$.

2. If 2 chords are congruent, then they are equidistant from the center.



$\overline{AB} \cong CD$ and $\overline{EG} \cong \overline{FG}$

3. If a diameter is perpendicular to a chord, then it bisects the chord also resulting in congruent arcs.

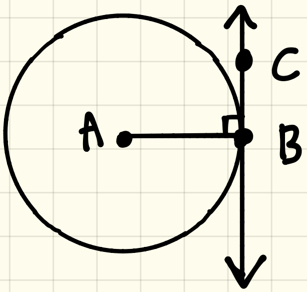


BE = diameter

$\overline{BE} \perp \overline{CD}$ and $\overline{CG} \cong \overline{DG}$.

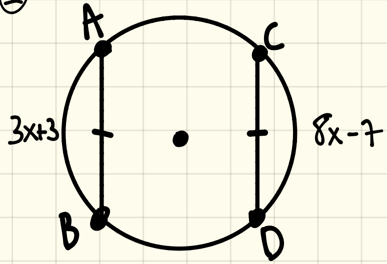
also $\overset{\frown}{CB} = \overset{\frown}{DB}$.

5. If a tangent intersects with a radius of a center, then the intersection forms a right angle.



[Examples] Find the unknown.

1. Find the arc measure.



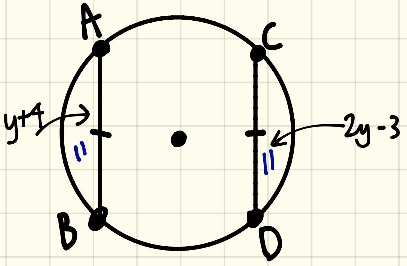
Since $\overline{AB} \cong \overline{CD}$, then $\overset{\frown}{AB} = \overset{\frown}{CD}$.

$$\begin{aligned}
\overset{\frown}{AB} &= \overset{\frown}{CD} \\
3x+3 &= 8x-7 \\
-3 &= \quad -3 \\
\hline
3x &= 8x-10 \\
-8x &= -8x \\
\hline
-5x &= -10 \\
x &= 2.
\end{aligned}$$

$$\begin{aligned}
\overset{\frown}{AB} &= 3x+3 \\
&= 3(2)+3 \\
&= 9^\circ
\end{aligned}$$

$$\begin{aligned}
\overset{\frown}{AB} &= \overset{\frown}{CD} \\
9^\circ &= \overset{\frown}{CD}.
\end{aligned}$$

2. Find $\overline{CD}$.



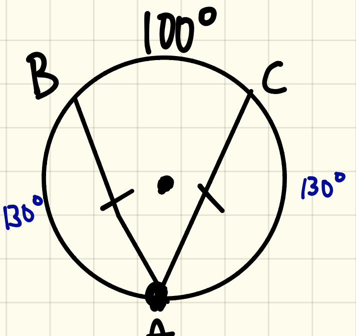
$\overline{AB} \cong \overline{CD}$

$$\begin{aligned}
y+4 &= 2y-3 \\
-4 &= \quad -4 \\
\hline
y &= 2y-7 \\
-2y &= -2y \\
\hline
-y &= -7 \\
y &= 7
\end{aligned}$$

$$\begin{aligned}
\overline{AB} &= y+4 \\
\overline{AB} &= 7+4 \\
&= 11
\end{aligned}$$

$$\begin{aligned}
\overline{AB} &= \overline{CD} \\
11 &= \overline{CD}
\end{aligned}$$

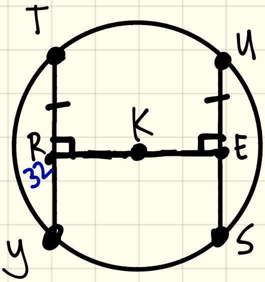
3. Find $\overset{\frown}{AB}$



Since $\overline{AB} \cong \overline{AC}$, then $\overset{\frown}{AB} = \overset{\frown}{AC}$

$$\begin{aligned}
\overset{\frown}{BC} + \overset{\frown}{AB} + \overset{\frown}{AC} &= 360^\circ \\
100^\circ + \overset{\frown}{AB} + \overset{\frown}{AB} &= 360^\circ \\
100^\circ + 2\overset{\frown}{AB} &= 360^\circ \\
2\overset{\frown}{AB} &= 260 \\
\overset{\frown}{AB} &= 130^\circ
\end{aligned}$$

3. Find length of $\overline{TY}$. if $\overline{US} = 4x$ and $\overline{TY} = -3x + 56$.

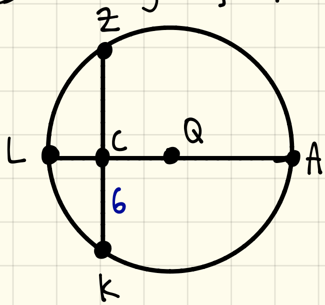


$\overline{TY} \cong \overline{US}$

$$\begin{aligned} -3x + 56 &= 4x \\ +3x \quad &= +3x \\ \hline 56 &= 7x \\ 8 &= x \end{aligned}$$

$$\begin{aligned} \overline{TY} &= -3x + 56 \\ &= -3(8) + 56 \\ &= 32 \end{aligned}$$

4. Find length of $\overline{CK}$ if $CK = 2x+3$ and $\overline{CZ} = 4x$.



$\overline{CZ} \cong \overline{CK}$

$$\begin{aligned} 2x + 3 &= 4x \\ -2x \quad &= -2x \\ \hline \frac{3}{2} &= \frac{\cancel{2}x}{\cancel{2}} \\ 1.5 \approx \frac{3}{2} &= x \end{aligned}$$

$$\begin{aligned} \overline{CK} &= 2x + 3 \\ &= 2(1.5) + 3 \\ &= 6. \end{aligned}$$