3.3 Sides (Angles) where vertex is outside & vertex is inside of Circle.

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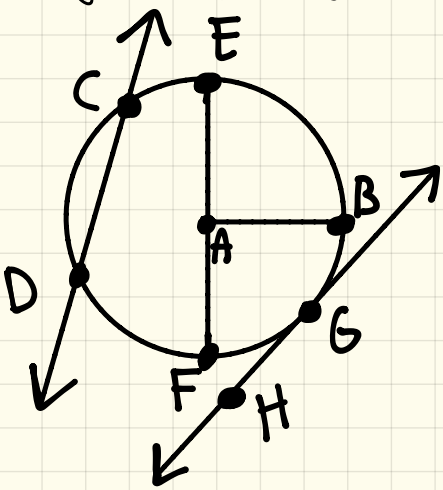
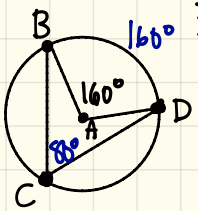
Old Circle Vocabulary

Identify the different types of segments & lines in the circle.



1. Center $\odot A$
2. Radius $\overline{AB}$
3. Diameter $\overline{EF}$
4. Chord $\overline{EF}$
5. Secant $\overleftrightarrow{CD}$
6. Tangent $\overleftrightarrow{GH}$
7. Minor Arc $\overset{\frown}{EB}$
8. Major Arc $\overset{\frown}{EFB}$
9. Semicircle $\overset{\frown}{EDF}$

Let's consider a circle where vertex meets inside the circle at the center of the circle.



Find $\overset{\frown}{BD}$, $m\angle BCD$.

$\overset{\frown}{BD}$ = intercepted arc
$\angle BAD$ = central angle
$\angle BCD$ = inscribed angle

Find $\overset{\frown}{BD}$.

$\overset{\frown}{BD} = \angle BAD$

$\overset{\frown}{BD} = 160°$

inscribed arc = $\frac{\text{central angle}}{2}$

Find $\angle BCD$.

$\angle BCD = \frac{\angle BAD}{2}$

$\angle BCD = \frac{160°}{2} = 80°$

Conclusion: Vertex that meets inside the circle at the center creates a central angle. The intercepted arc & central angle have the relationship of: central angle = intercepted arc.

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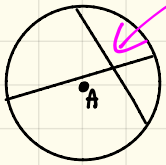
NEW-A Angles form from vertex inside or outside Circle

Let's consider a circle where vertex meets inside the circle NOT at the center of the circle.



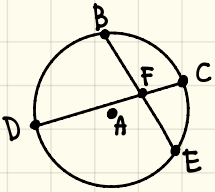
vertex does not meet at the circle.

Is there a relationship between the angle inside the circle & the intercepted arc?

Dilemma: Find a relationship between an angle formed by a vertex not in the center & its intercepted arc.

case 1 (INSIDE) ⇒ Add!

Vertex is inside the Circle (not at the center)

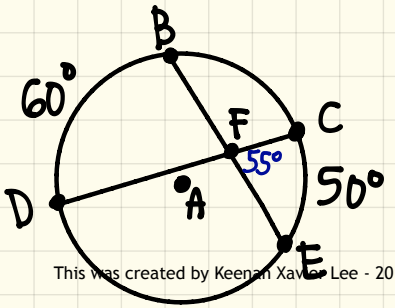


$$\frac{\text{Intercepted Arc} + \text{Intercepted Arc}}{2} = \text{Angle of Interest}$$

$$\frac{\overset{\frown}{BC} + \overset{\frown}{DE}}{2} = m\angle BFC$$

[Examples] Find the unknown(s).

① Find $m\angle CFE$.



$$\frac{\overset{\frown}{BD} + \overset{\frown}{CE}}{2} = m\angle CFE$$

$$\frac{50^\circ + 60^\circ}{2} = m\angle CFE$$

$$55^\circ = \frac{110^\circ}{2} = m\angle CFE$$

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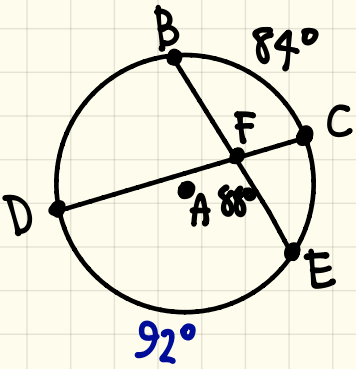
② Find $\overset{\frown}{DE}$.



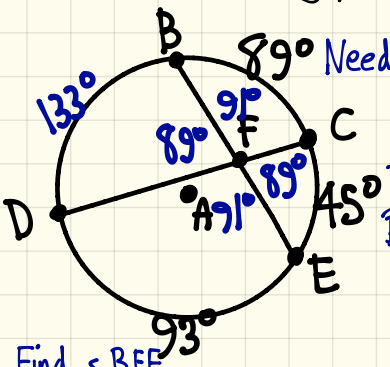
$$\frac{\overset{\frown}{DE} + \overset{\frown}{BC}}{2} = \angle DFE$$

$$\frac{\overset{\frown}{DE} + 84^\circ}{2} = 88^\circ$$

$$\overset{\frown}{DE} + 84^\circ = 176^\circ$$

$$\overset{\frown}{DE} = 92^\circ.$$

③ Find all the missing parts of the circle.



Need to find $\overset{\frown}{BD}$, $\angle CFE$, $\angle DFE$, $\overset{\frown}{BFD}$, $\angle BFC$.

Find $\overset{\frown}{BD}$.

$\overset{\frown}{BD} + \overset{\frown}{BC} + \overset{\frown}{CE} + \overset{\frown}{DE} = 360^\circ$

$\overset{\frown}{BD} + 89^\circ + 45^\circ + 93^\circ = 360^\circ$

$\overset{\frown}{BD} + 227^\circ = 360^\circ$

$\overset{\frown}{BD} = 133^\circ$

Find $\angle CFE$.

$$\angle CFE = \frac{\overset{\frown}{BD} + \overset{\frown}{CE}}{2}$$

$$\angle CFE = \frac{133^\circ + 45^\circ}{2}$$

$$\angle CFE = 89^\circ$$

Find $\angle BFE$.

$\angle BFE = \angle CFE$ because they are vertical angles.

$\angle BFE = 89^\circ$

Find $\angle BFC$.

$\angle BFC + \angle CFE = 180^\circ$ because they are a linear pair.

$\angle BFC + 89^\circ = 180^\circ$

$\angle BFC = 91^\circ$

Find $\angle DFE$.

$\angle DFE = \angle BFC$ because they are vertical angles.

$\angle DFE = 91^\circ$

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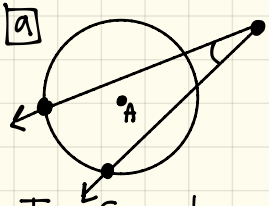
case 2 (OUTSIDE) ⇒ Subtract!

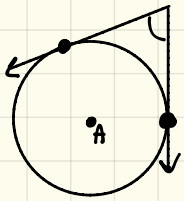
Vertex is outside the Circle (not at the center)

a



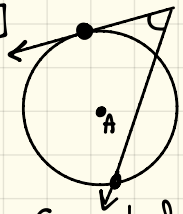
Two Secants

b



Two Tangents

c

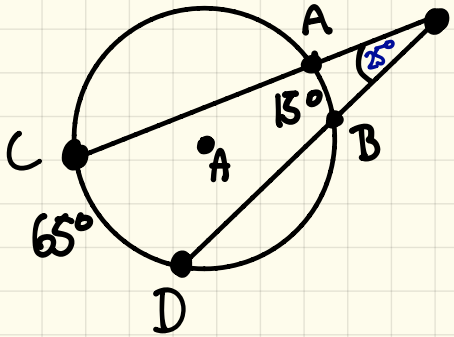


One Secant & One Tangent

$$\frac{\text{Bigger Arc} - \text{Smaller Arc}}{2} = \text{Outside Angle}$$

[Examples] Find the unknown.

① Find $\angle AEB$.



$$\text{Outside Angle} = \frac{\text{Big Arc} - \text{Small Arc}}{2}$$

$$\angle AEB = \frac{\overset{\frown}{CD} - \overset{\frown}{AB}}{2}$$

$$\angle AEB = \frac{65° - 15°}{2}$$

$$\angle AEB = 25°$$

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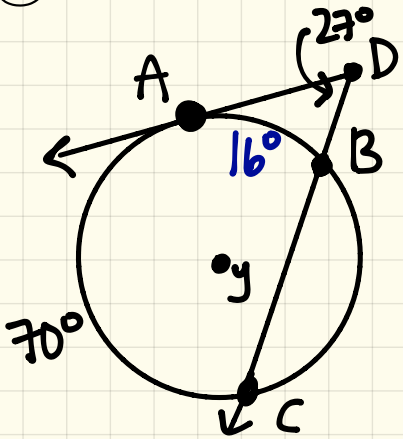
② Find $\overset{\frown}{AB}$.



$$\frac{\text{Big Arc} - \text{Small Arc}}{2} = \text{Outside Angle}$$

$$\frac{\overset{\frown}{AC} - \overset{\frown}{AB}}{2} = m\angle ADB$$

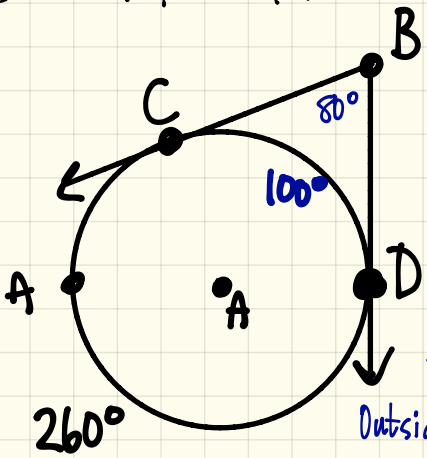
$$\frac{70° - \overset{\frown}{AB}}{2} = 27°$$

$$70° - \overset{\frown}{AB} = 54°$$

$$-\overset{\frown}{AB} = -16°$$

$$\overset{\frown}{AB} = 16°$$

③ Find all parts of the unknowns in the circle.



Need to find $m\angle CBD$ & $\overset{\frown}{CD}$.

Find $\overset{\frown}{CD}$.

$$\overset{\frown}{CD} + \overset{\frown}{CAD} = 360°$$

$$\overset{\frown}{CD} + 260° = 360°$$

$$\overset{\frown}{CD} = 100°$$

Find $m\angle CBD$.

$$\text{Outside Angle} = \frac{\text{Big Arc} - \text{Small Arc}}{2}$$

$$m\angle CBD = \frac{\overset{\frown}{CAD} - \overset{\frown}{CD}}{2}$$

$$m\angle CBD = \frac{260° - 100°}{2}$$

$$m\angle CBD = 80°$$

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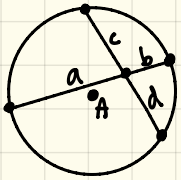
new-B Segments form from vertex inside or outside Circle

Let's talk about segment measures...

case 1

Vertex is inside the Circle (not at the center)

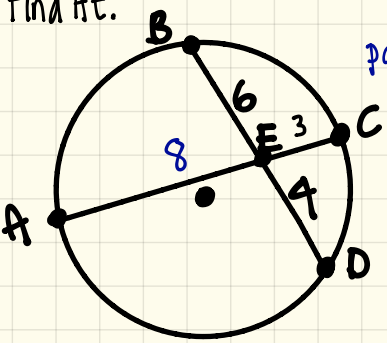


part * part = part * part

$a * b = c * d$

[Examples] Find the unknown.

Find $\overline{AE}$.



part * part = part * part

$\overline{AE} * \overline{CE} = \overline{BE} * \overline{DE}$

$\overline{AE} * 3 = 6 * 4$

$3\overline{AE} = 24$

$\overline{AE} = 8.$

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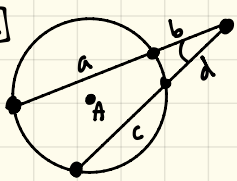
case 2

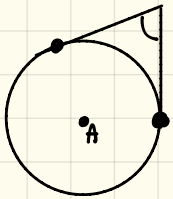
Vertex is outside the Circle (not at the center)

a



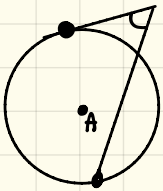
Two Secants

b



Two Tangents

c



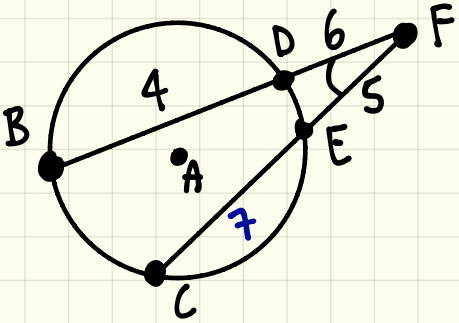
One Secant & One Tangent

Outside * whole = outside * whole

$b * (a+b) = d * (c+d)$

[Examples] Find the unknown.

① Find $\overline{CE}$.



Outside * whole = outside * whole

$\overline{DF} * \overline{BF} = \overline{EF} * \overline{CF}$

$6 * 10 = 5 * (5+\overline{CE})$

$60 = 5(5+\overline{CE})$

$60 = 25+5\overline{CE}$

$35 = 5\overline{CE}$

$7 = \overline{CE}$

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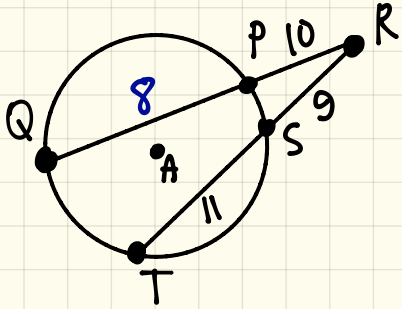
② Find $\overline{PR}$.



outside * whole = outside * whole

$\overline{PR} * \overline{QR} = \overline{RS} * \overline{RT}$

$10 * (10 + \overline{QP}) = 9 * 20$

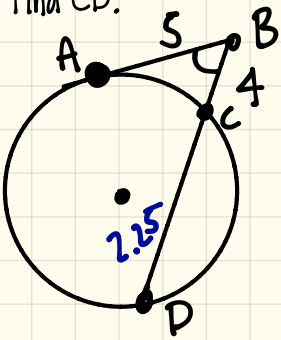
$10(10 + \overline{QP}) = 180$

$100 + 10\overline{QP} = 180$

$10\overline{QP} = 80$

$\overline{QP} = 8$

③ Find $\overline{CD}$.



outside x whole = outside * whole

$5 * 5 = 4 * (4 + \overline{CD})$

$25 = 4(4 + \overline{CD})$

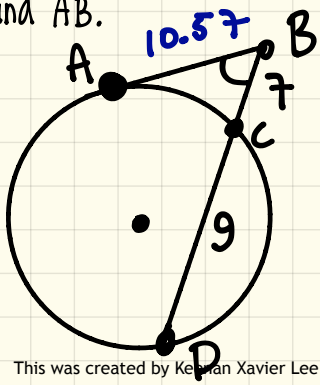
$25 = 16 + 4\overline{CD}$

$9 = 4\overline{CD}$

$2.25 = \frac{9}{4} = \overline{CD}$

Note: Outside & whole are the same in tangents.

④ Find $\overline{AB}$.



outside x whole = outside * whole

$\overline{AB} * \overline{AB} = \overline{BC} * \overline{BD}$

$\overline{AB}^2 = 7 * 16$

$\overline{AB}^2 = 112$

$\sqrt{\overline{AB}^2} = \sqrt{112}$

$AB = \sqrt{112} = 4\sqrt{7} \approx 10.58$

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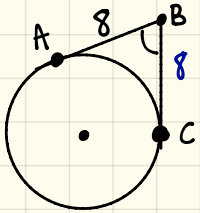
⑤ Find $\overline{BC}$.



outside * whole = outside * whole

$\overline{AB} * \overline{AB} = \overline{BC} * \overline{BC}$

$8 * 8 = \overline{BC} * \overline{BC}$

$64 = \overline{BC}^2$

$\sqrt{64} = \sqrt{\overline{BC}^2}$

$8 = \overline{BC}$.

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