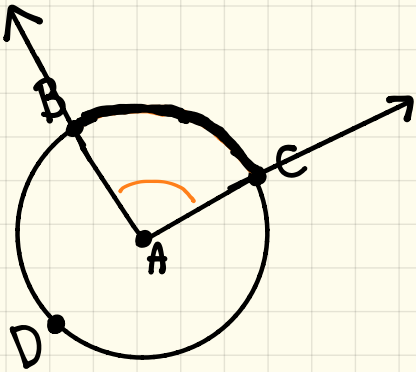
3.2 Inscribed Angles, Quadrilaterals

Old Central Angle

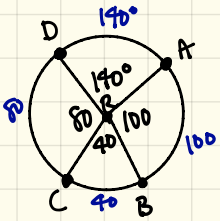
Recall: A central angle is an angle in between the intercepted arc.



Let's take intercepted arc $\overset{\frown}{BC}$.
The central angle is $\angle BAC$.

Central Angle = Intercepted Arc

[Example] Determine the measure of the arcs.

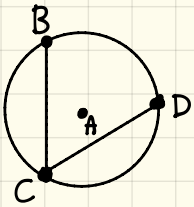


Find $m\overset{\frown}{DAB}$.
Find $m\overset{\frown}{BCA}$.

$m\overset{\frown}{DAB} = \overset{\frown}{DA} + \overset{\frown}{AB}$
$= 140° + 100°$
$= 240°$

$m\overset{\frown}{BCA} = \overset{\frown}{BC} + \overset{\frown}{CD} + \overset{\frown}{DA}$
$= 40° + 80° + 140°$
$= 260°$

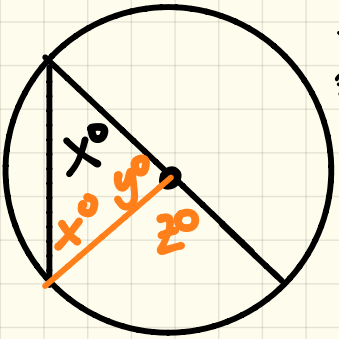
new Inscribed Angles



Inscribed Angles $< BCD$.
↳ Inscribed angle is an angle created by 2 chords meeting at the vertex on the circle.

Let's consider the below diagram. Find $x°$.

Recall: Base Angle Thm.



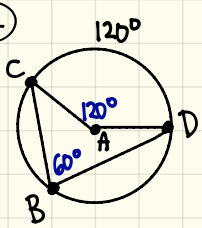
$x°$ = inscribed angle
$z°$ = central angle.

(Triangle sum Thm.) $x + x + y = 180°$
(linear pair) $y + z = 180°$

$$x + x + y = y + z$$
$$2x + y = y + z$$
$$2x + y = y + z$$
$$\frac{2x}{2} = \frac{z}{2}$$
$$x = \frac{z}{2}$$

Inscribed Angle = $\frac{\text{Central Angle}}{2}$

[Examples] Find the inscribed angle.

① 

Find m<CBD.

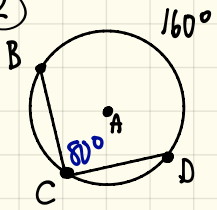
need to find m<CAD: (central angle)
$\overset{\frown}{CD}$ is intercepted arc & m<CAD is the central angle
m<CAD = 120°

(Inscribed angle)
need to m<CBD:
Inscribed angle = $\frac{\text{central angle}}{2}$

$m<CBD = \frac{m<CAD}{2}$

$m<CBD = \frac{120°}{2}$

m<CBD = 60°

② 

Find m<BCD.

Inscribed angle = $\frac{\text{central angle}}{2}$

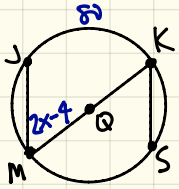
Although there isn't a central angle given, the intercepted arc $\overset{\frown}{BD}$ is given. Remember the intercepted arc = central angle.

$m<BCD = \frac{\overset{\frown}{BD}}{2}$

$m<BCD = \frac{160°}{2}$

$m<BCD = 80°$

③ If $\overset{\frown}{JK} = 80°$ and $m\angle JMK = 2x - 4$. Find x.



$\overset{\frown}{JK}$ = intercepted arc (equals central angle)
$m\angle JMK$ = inscribed angle

$$\text{inscribed angle} = \frac{\text{central angle}}{2}$$

$$m\angle JMK = \frac{\overset{\frown}{JK}}{2}$$

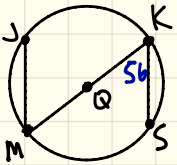
$$2x - 4 = \frac{80}{2}$$

$$2x - 4 = 40$$

$$2x = 44$$

$$x = 22$$

④ If $m\angle MKS = 56°$. Find $\overset{\frown}{MS}$.



$m\angle MKS$ = inscribed angle
$\overset{\frown}{MS}$ = intercepted arc (equals central angle)

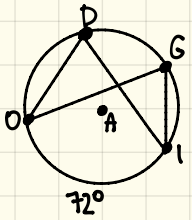
$$\text{inscribed angle} = \frac{\text{central angle}}{2}$$

$$m\angle MKS = \frac{\overset{\frown}{MS}}{2}$$

$$56° = \frac{\overset{\frown}{MS}}{2}$$

$$112° = \overset{\frown}{MS}.$$

⑤ Find m< ODI and m< OGI



Find m< ODI
$\overset{\frown}{OI}$ = intercepted arc (equals central angle)
m< ODI = inscribed angle

$$\text{inscribed angle} = \frac{\text{central angle}}{2}$$

$$m< ODI = \frac{\overset{\frown}{OI}}{2}$$

$$m< ODI = \frac{72}{2}$$

$$m< ODI = 36°$$

Find m< DIG.
$\overset{\frown}{OI}$ = intercepted arc (equals central angle)
m< OGI = inscribed angle

$$\text{inscribed angle} = \frac{\text{central angle}}{2}$$

$$m< OGI = \frac{\overset{\frown}{OI}}{2}$$

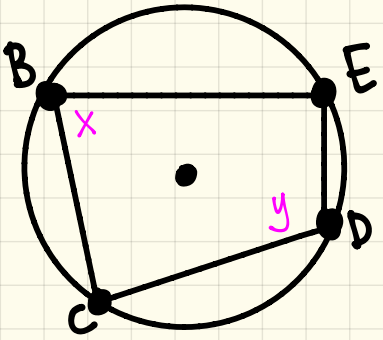
$$m< OGI = \frac{72}{2}$$

$$m< OGI = 36°$$

Note: If 2 inscribed angles share the same intercepted arc, then the 2 inscribed angles are equal.

Let's consider a circle with an inscribed quadrilateral where all 4 vertices lie on the circle.

$$\frac{\text{Central Angle}}{2} = \text{Inscribed Angle}$$



$\frac{\overset{\frown}{CDE}}{2} = x$ $\qquad$ $\frac{\overset{\frown}{CBE}}{2} = y$

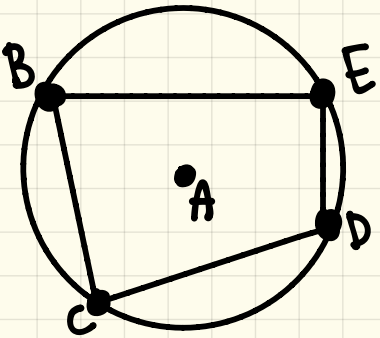
$\overset{\frown}{CDE} = 2x$ $\qquad$ $\overset{\frown}{CBE} = 2y$

$$\overset{\frown}{CDE} + \overset{\frown}{CBE} = 360°$$
$$2x + 2y = 360°$$
$$2(x + y) = 360°$$
$$x + y = 180°$$

conclusion: Opposite angles are supplementary.

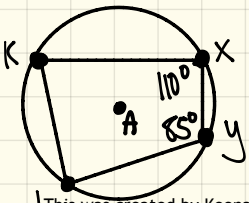
Quadrilaterals inscribed in a circle:

When Quadrilaterals are inscribed in a circle, opposite angles are supplementary.



$m\angle B + m\angle D = 180°$
$m\angle C + m\angle E = 180°$

[Example] Find all missing angles.



Find $m\angle L$

$m\angle L + m\angle X = 180°$
$m\angle L + 110° = 180°$
$m\angle L = 70°$

Find $m\angle K$

$m\angle K + m\angle y = 180°$
$m\angle K + 85° = 180°$
$m\angle K = 95°$