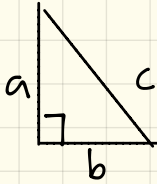


2.9 Trigonometric Ratios

Finding missing sides & missing angles using Right Triangle Trigonometry.

Old Pythagorean Theorem

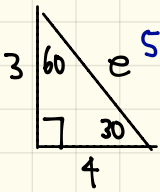
Recall: If there is a right triangle, then there is a relationship between the legs & hypotenuse which is $a^2 + b^2 = c^2$.



$$a^2 + b^2 = c^2$$

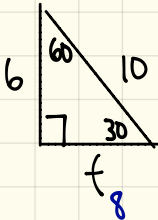
[Examples] Find the missing side.

①



$$\begin{aligned} a^2 + b^2 &= c^2 \\ (3)^2 + (4)^2 &= e^2 \\ 9 + 16 &= e^2 \\ 25 &= e^2 \\ \sqrt{25} &= \sqrt{e^2} \\ 5 &= e \end{aligned}$$

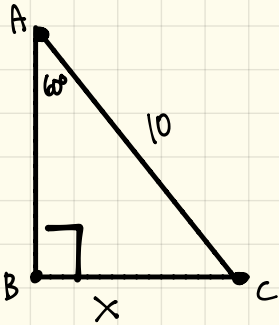
②



$$\begin{aligned} a^2 + b^2 &= c^2 \\ (6)^2 + f^2 &= 10^2 \\ 36 + f^2 &= 100 \\ f^2 &= 64 \\ \sqrt{f^2} &= \sqrt{64} \\ f &= 8 \end{aligned}$$

new Trigonometric Ratios

Let's consider the below right triangle. Solve for the unknown.



We can't use Pythagorean Theorem because we have to use 2 legs to find the hypotenuse.

Dilemma: How do we find the unknown sides when only given one side in a right triangle?

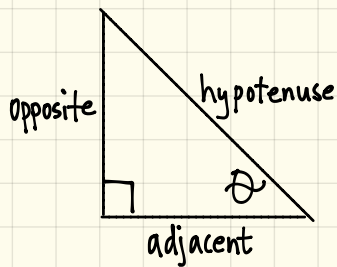
- The way we can find unknown sides in a right is by using Trigonometric Ratios.
- The Trigonometric Functions we will be looking at are:

<u>sine</u>	<u>cosine</u>	<u>tangent</u>
↳ pronounced "sign"	↳ pronounced "co-sign"	↳ pronounced "tan-gent"
↳ symbol $\sin \theta$	↳ symbol $\cos \theta$	↳ $\tan \theta$

- Greek letter θ → pronounced "theta" representing an unknown angle. (like x represents an unknown number)

TRIG RATIOS

Memorize



$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

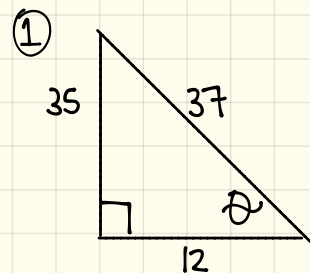
$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

- "opposite" means opposite side of θ
- "adjacent" means next to θ
- "hypotenuse" means longest or diagonal side.

A good way to memorize is SOH-CAT-TOA! Sounds like an Indian word :)

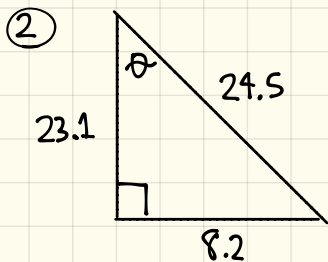
[Examples] Find $\sin \theta$, $\cos \theta$ & $\tan \theta$ from the right triangles.



$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{35}{37}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{12}{37}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{35}{12}$$

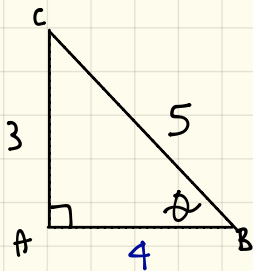


$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{8.2}{24.5}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{23.1}{24.5}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{8.2}{23.1}$$

③ Consider $\triangle ABC$ where $m\angle BAC = 90^\circ$ and $\sin \theta = \frac{3}{5}$.
Find AB and $\tan \theta$?



$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{3}{4}$$

$$a^2 + b^2 = c^2$$

$$3^2 + b^2 = 5^2$$

$$9 + b^2 = 25$$

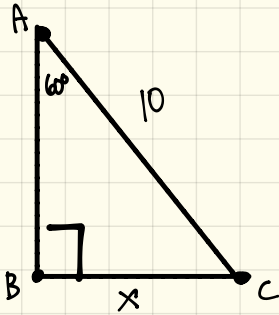
$$b^2 = 16$$

$$\sqrt{b^2} = \sqrt{16}$$

$$b = 4.$$

How do we use trig ratios to solve for missing sides?

Let's go back to the our dilemma.



- What's the angle of interest? 60°
- What's the position of side length x in respect to 60° ? "opposite"
- What is the position of side length 10 in respect to 60° ? "hypotenuse"

Which trig ratio is appropriate to use? $\sin \theta$.

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\Rightarrow \sin 60^\circ = \frac{x}{10}$$

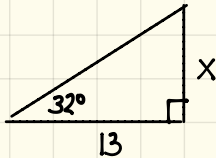
Solve for x by cross multiplication.

$$x = 10 \sin 60^\circ$$

$$x = 5\sqrt{3} \approx 8.66$$

[More Examples] Find the missing side.

①

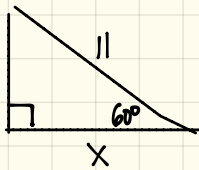


$$\tan \theta = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan 32^\circ = \frac{x}{13}$$

$$13 \tan 32^\circ = x$$

$$8.12 \approx x$$

②



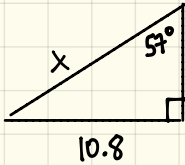
$$\cos \theta = \frac{\text{adj}}{\text{hyp}} \Rightarrow \cos 60^\circ = \frac{x}{11}$$

$$11 \cos 60^\circ = x$$

$$\frac{11}{2} = x$$

$$5.5 = x$$

③

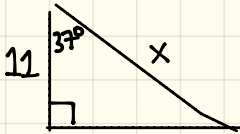


$$\sin \theta = \frac{\text{opp}}{\text{hyp}} \Rightarrow \sin 57^\circ = \frac{10.8}{x}$$

$$\frac{x \sin 57^\circ}{\sin 57^\circ} = \frac{10.8}{\sin 57^\circ}$$

$$x \approx 12.88$$

④

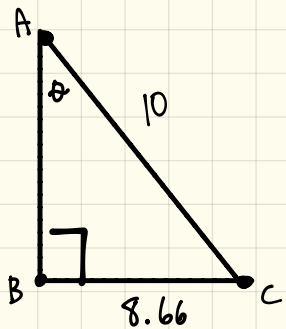


$$\cos \theta = \frac{\text{adj}}{\text{hyp}} \Rightarrow \cos 37^\circ = \frac{11}{x}$$

$$\frac{x \cos 37^\circ}{\cos 37^\circ} = \frac{11}{\cos 37^\circ}$$

$$x \approx 13.77$$

Let's consider a triangle with 2 given sides & a missing angle:



$$\sin \theta = \frac{\text{opp}}{\text{adj}} \Rightarrow$$

$$\sin \theta = \frac{8.66}{10}$$

$$\sin^{-1}\left(\frac{8.66}{10}\right) = \theta$$

$$59.99^\circ \approx \theta$$

$$60^\circ \approx \theta$$

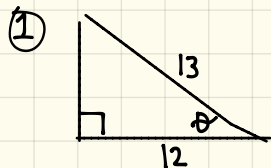
Now, I'm trying to find θ . \Rightarrow We need to isolate θ by using INVERSE OPERATIONS.

• The inverse of $\sin \theta$ is $\sin^{-1}\left(\frac{\text{opp}}{\text{hyp}}\right)$.

• The inverse of $\cos \theta$ is $\cos^{-1}\left(\frac{\text{adj}}{\text{hyp}}\right)$.

• The inverse of $\tan \theta$ is $\tan^{-1}\left(\frac{\text{opp}}{\text{adj}}\right)$.

[More Examples] Find the missing angle.



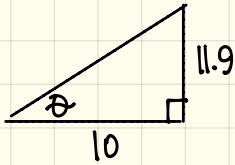
$$\cos \theta = \frac{\text{adj}}{\text{hyp}} \Rightarrow \cos \theta = \frac{12}{13}$$

$$\cos^{-1}\left(\frac{12}{13}\right) = \theta$$

$$22.61^\circ \approx \theta$$

$$23^\circ \approx \theta$$

②



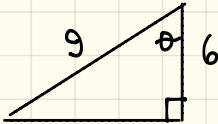
$$\tan \theta = \frac{\text{opp}}{\text{adj}} \Rightarrow \tan \theta = \frac{11.9}{10}$$

$$\tan^{-1}\left(\frac{11.9}{10}\right) = \theta$$

$$49.95^\circ \approx \theta$$

$$50^\circ \approx \theta$$

③



$$\cos \theta = \frac{\text{adj}}{\text{hyp}} \Rightarrow \cos \theta = \frac{6}{9}$$

$$\cos^{-1}\left(\frac{6}{9}\right) = \theta$$

$$48.2^\circ \approx \theta$$