2.3 Triangle Congruence

Old Isometric Transformations

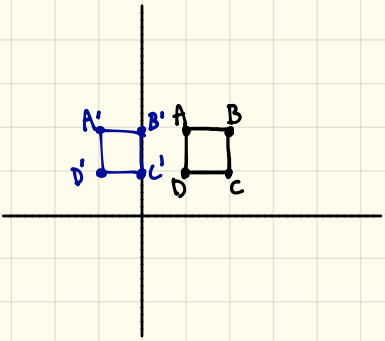
Let's recall the isometric transformations:

① Translations (slide)

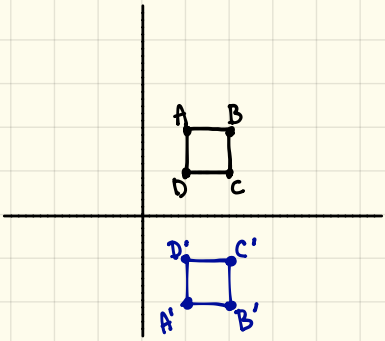
② Reflections (flip)

③ Rotations (turn)

① $(x, y) \rightarrow (x-2, y)$



② $(x, y) \rightarrow (x, -y)$

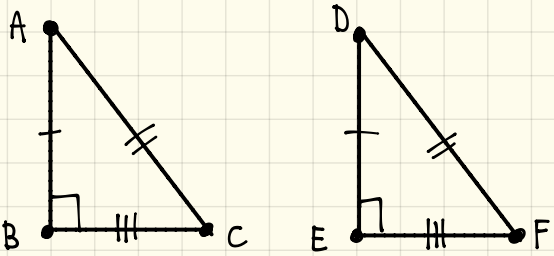


- Preimage to the image is the same shape & same size.

Isometric transformations (rigid motion) is where the distances between the points are preserved. Basically, the image is congruent to its preimage.

[New] Triangle Congruence

Let's consider the following 2 triangles:

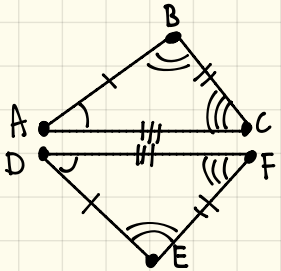


$\Delta ABC \cong \Delta DEF$ ← note: order of the statement matters

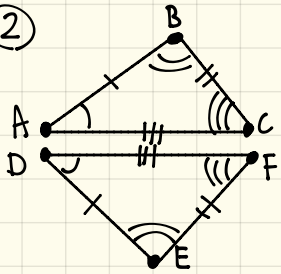
- Congruent Triangles must have 3 congruent sides & 3 congruent angles
- The parts of congruent triangles that "match" are called corresponding parts.
- In a congruent statement, ORDER MATTERS! Everything must match up.

Sides	Angles
$\overline{AB} \cong \overline{DE}$	$\angle A \cong \angle D$
$\overline{BC} \cong \overline{EF}$	$\angle B \cong \angle E$
$\overline{AC} \cong \overline{DF}$	$\angle C \cong \angle F$

[Examples] Complete each congruence statement.

① 

If $\Delta ABC \cong \Delta DEF$, then $\overline{BC} \cong \overline{EF}$

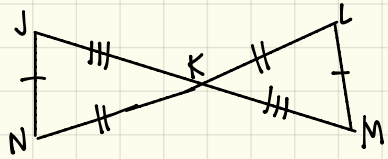
② 

If $\triangle ABC \cong \triangle DEF$,
then $\angle C \cong \angle F$

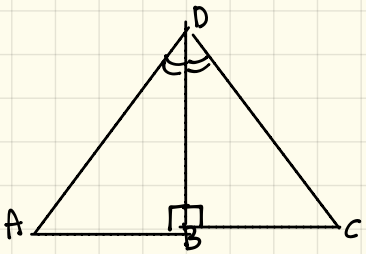
③ $\triangle CAT \cong \triangle DDG$
then $\overline{AC} \cong \overline{OD}$

④ $\triangle BAT \cong \triangle MON$
$\angle T \cong \angle N$
$\angle ATB \cong \angle ONM$
$\overline{BA} \cong \overline{MO}$
$\overline{NM} \cong \overline{TB}$

⑤ $\triangle BCA \cong \triangle EGF$
$\triangle CAB \cong \triangle GFE$

⑥ 

$\triangle MKL \cong \triangle JKN$

⑦ 

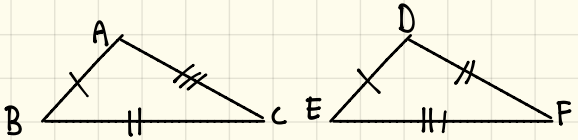
$\triangle ABD \cong \triangle CBD$

How do we prove 2 triangles are congruent (meaning 3 corresponding congruent sides & corresponding congruent angles).

To prove triangle congruence, you will need to check for congruency in 3 specific components.

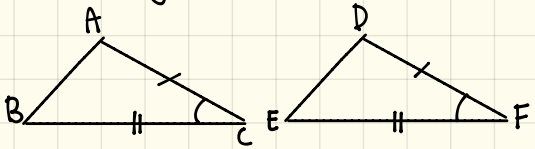
There are 5 ways to prove triangles congruent.

1. Side-Side-Side (SSS)



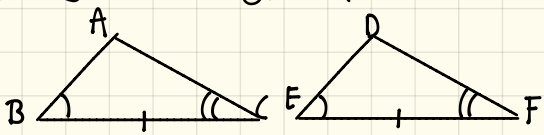
All three sides in one triangle are congruent to the corresponding three sides in the other triangle.

2. Side-Angle-Side (SAS)



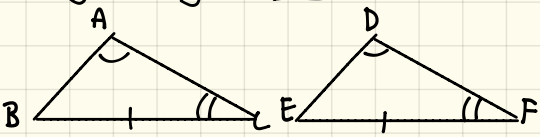
Two sides & the INCLUDED angle in one triangle are congruent to the corresponding two sides & INCLUDED side in other triangle (the angle is in between the 2 marked sides)

3. Angle-Side-Angle (ASA)



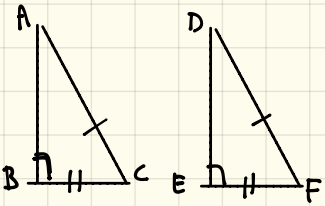
Two angles & the INCLUDED sides in one triangle are congruent to the corresponding two angles & INCLUDED side in other triangle (the angle is in between the 2 marked sides)

4. Angle-Angle-Side (AAS)



Two angles and one side that is NOT included in one triangle is congruent to the corresponding two angles and one side NOT included in the other triangle.

5. Hypotenuse-Leg (HL)



Must be right triangles where the hypotenuse & one leg in one triangle is congruent to the hypotenuse & one leg in the other triangle.

5 Ways to Prove Triangles are Congruent:

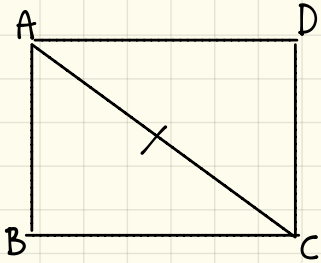
SSS
SAS
ASA
AAS
HL

These are NOT ways to prove Triangle Congruence:

ASS
SSA
AAA

note: 2 markings you can add if they notated already

Share a Side (Rectangle)

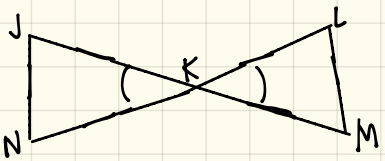


Reason: Reflexive Property

$\Delta ABC \cong \Delta ADC$

$\overline{AC} \cong \overline{AC}$

Vertical Angles



Reason: Vertical Angles are Congruent.

$\angle JKN \cong \angle LKM$