2.1 Vertical Angles, Linear Pair, Complementary Angles & Angle Bisectors

Old Angles

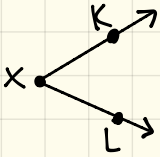
Let's recall the different types of angles.

a **Acute Angles**
has an angle measuring between 0° and 90°.

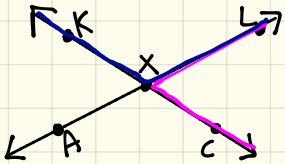
b **Right Angles**
has an angle measuring exactly 90°.

c **Obtuse Angles**
has an angle measuring between 90° and 180°

Also, let's recall the notation of angles.



∠X ← angle x.

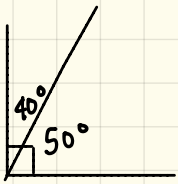


- ∠KXL ← top angle X
- ∠LXC ← right angle x

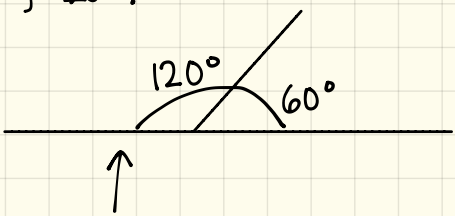
new Complementary & Supplementary Angles

Complementary Angles
Pairs of angles that sum to 90°.

(Example)

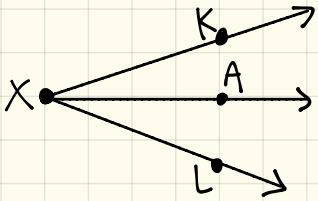


Supplementary Angles
Pairs of angles that sum of 180°.



- straight line angle adds up to 180°
- linear pair is 60° & 120°.

Angle Bisector: A ray (or line or segment) that divides an angle into 2 congruent angles (2 angles with equal measures).

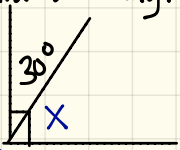


· $\overrightarrow{XA}$ is an angle bisector

$\angle KXA \cong \angle AXL$ so $m\angle KXA = m\angle AXL$.

[Examples] Answer the following.

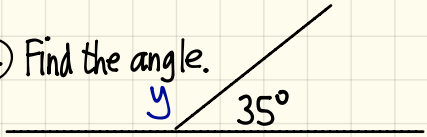
① Find the angle.



$x + 30° = 90°$

$x = 60°$

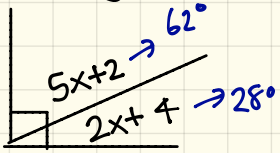
② Find the angle.



$y + 35° = 180°$

$y = 145°$

③ Find the angle.



$5x + 2 + 2x + 4 = 90°$

$5x + 2x + 2 + 4 = 90°$

$7x + 6 = 90°$

$-6 = -6$

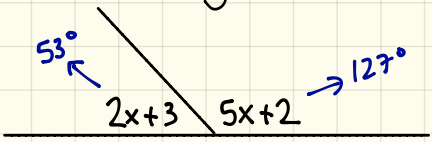
$\frac{7x}{7} = \frac{84}{7}$

$x = 12$

$5(12)+2 = 62°$

$2(12)+4 = 28°$

④ Find the angle.



$5x + 2 + 2x + 3 = 180°$

$5x + 2x + 2 + 3 = 180°$

$7x + 3 = 180°$

$-6 = -6$

$\frac{7x}{7} = \frac{175}{7}$

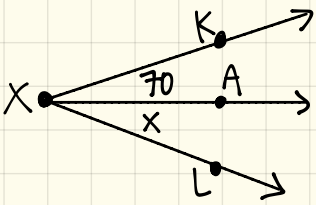
$x = 25$

$5(25)+2 = 127$

$2(25)+3 = 53$

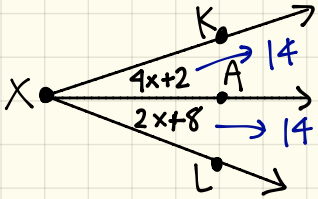
[Examples] Find the missing angles.

⑤ $\overrightarrow{XA}$ is an angle bisector.



$x = 70°$

⑥ $\overrightarrow{XA}$ is an angle bisector.



$2x+8 = 4x+2$

$-8 = -8$

$2x = 4x-6$

$-4x = -4x$

$-2x = -6$

$\frac{-2x}{-2} = \frac{-6}{-2}$

$x = 3$

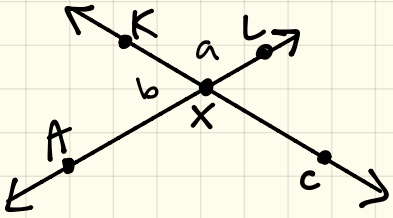
$2(3)+8 = 14°$

$\angle KXA = 14°$

$\angle AXL = 14°$

Vertical Angles

Let's consider the below diagram.



$\angle KXL = a^\circ$
$\angle KXA = b^\circ$

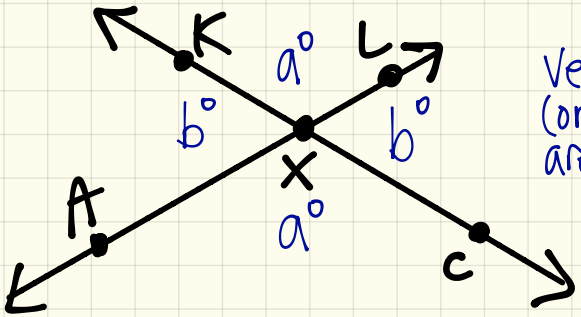
Find the missing angles.

- Since a & b are a linear pair, they are to 180° $(a+b=180^\circ)$.

What about $\angle AXC$ & $\angle LXC$?

$\angle AXC$ is a linear pair to $\angle KXA$.
So, $\angle AXC + b = 180^\circ$ $(a = 180 - b)$
Therefore, $\angle AXC = a$.

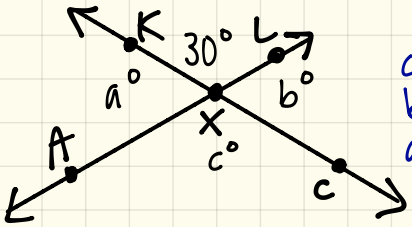
$\angle LXC$ is a linear pair to $\angle KXL$.
So, $\angle LXC + a = \angle KXL$.
Therefore, $\angle LXC = b$.



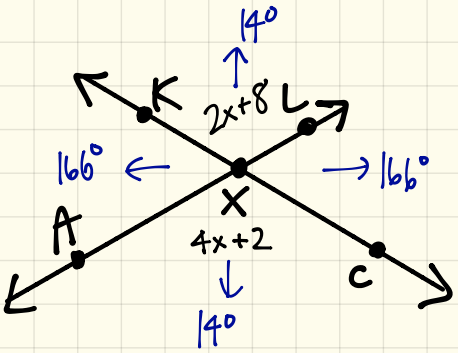
Vertical angles (or opposite angles) are congruent.

[Example] Find missing angles.

Given $\angle KXL = 30°$.



$c = 30° \rightarrow \angle KXL = \angle AXC$ are vertical angles.
$b = 150° \rightarrow \angle KXL + \angle LXC$ are linear pair
$a = 150° \rightarrow \angle KXL + \angle KXA$ are linear pair
or $\angle KXA = \angle LXC$ are vertical angles.



Given $\angle KXL = 2x+8$ & $\angle AXC = 4x+2$

$$4x+2 = 2x+8$$
$$-2 = -2$$
$$4x = 2x+6$$
$$-2x = -2x$$
$$\frac{2x}{2} = \frac{6}{2}$$
$$x = 3.$$

$2(3)+8 = 14°$

$\angle KXL = 14°$
$\angle AXC = 14°$
$\angle KXA = 166°$
$\angle LXC = 166°$