

1.9 Solving Quadratics

Part 1

GCF Factoring, Differences of Squares, Factoring Trinomials

Old-A Factoring

Factor the expressions.

$$\textcircled{1} x^2 - 7x - 18 \quad \begin{matrix} 1 \cdot 18 \\ 2 \cdot 9 \end{matrix}$$
$$= (x-2)(x-9)$$

$$\textcircled{2} 7k^2 + 9k$$
$$= k(7k+9)$$

GCF of #'s
7 9
 $\textcircled{1}7$ $\textcircled{1}9$

GCF of variable
 k^2 k
 $\textcircled{k} \cdot k$ \textcircled{k}

$$\textcircled{3} p^2 - 5p - 14 \quad \begin{matrix} 1 \cdot 14 \\ 7 \cdot 2 \end{matrix}$$
$$= (p-7)(p+2)$$

$$\textcircled{4} 7x^2 - 31x - 20 \quad \begin{matrix} 1 \cdot 20 \\ 2 \cdot 10 \\ 4 \cdot 5 \end{matrix}$$
$$= (7x - 5)(x - 4)$$

(Note: A diagram shows the FOIL process with terms $7x^2$, $-28x$, $-5x$, and -20 being combined to form the factors.)

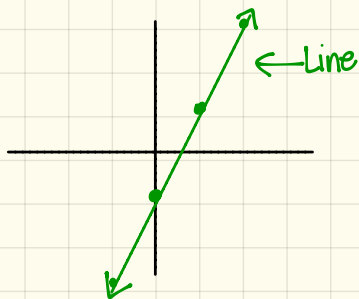
$$\textcircled{5} x^2 - 49$$
$$= (x-7)(x+7)$$

$$\textcircled{6} 2m^2 - 18m + 16$$
$$2(m^2 - 9m + 8) \quad \begin{matrix} 1 \cdot 8 \\ 2 \cdot 4 \end{matrix}$$
$$2(m-8)(m-1)$$

Old-B

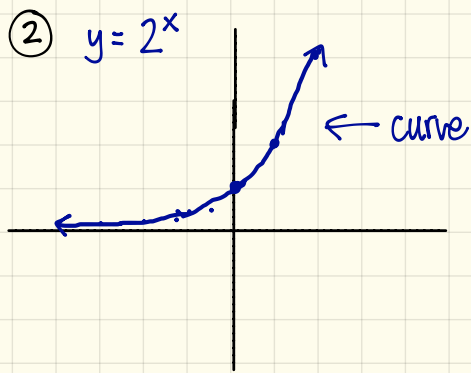
Linear & Exponential Functions (Algebraic / Graphic)

$$\textcircled{1} y = 2x - 1 \quad m=2, b=-1$$



x	y
-2	-5
-1	-3
0	-1
1	1
2	3
⋮	⋮

Parent Function of Linear Function
 $y = mx + b$

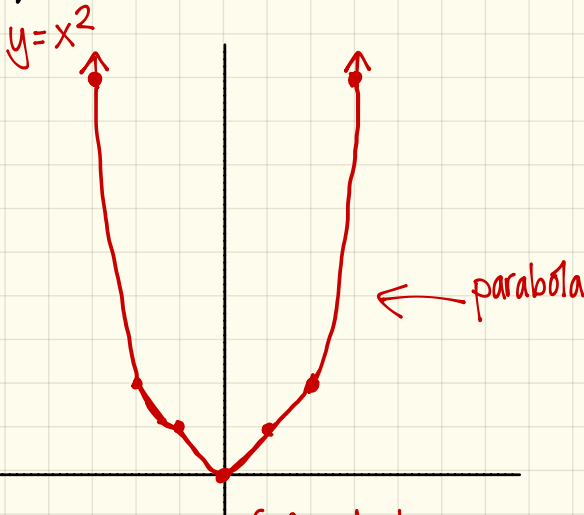


x	y
0	1
1	2
2	4
3	8
4	16
⋮	⋮

Parent Function of Exponentials
 $y = a^x$

new-A Quadratics

Let's consider the equation $y = x^2$. Create a graph & table of the equation.



x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9
⋮	⋮

Parent Function of Quadratics
 $y = x^2$

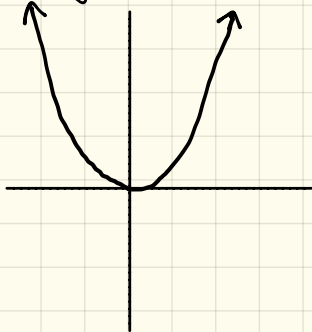
new-B Solving Quadratics

What does it mean to solve Quadratics?

Solving Quadratics means to find the x-intercepts of the Quadratic function.

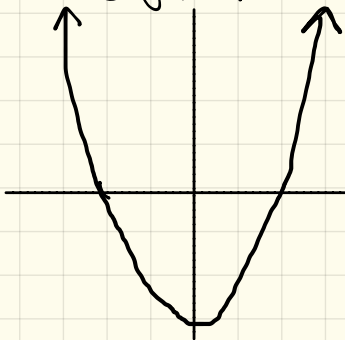
[Examples] Solve the Quadratics (Graphically)

① $y = x^2$



x-intercept: $(0, 0)$

② $y = x^2 - 4$



x-intercepts: $(-2, 0), (2, 0)$

③ $y = x^2 - 8x + 15$



x-intercepts: $(3, 0), (5, 0)$

Solving Quadratics Algebraically

Let's consider the graph of the equation of $y = x^2 - 8x + 15$.

How do we find the x-intercepts of this graph algebraically?

Finding the intersecting points between the 2 lines $y = x^2 - 8x + 15$ & $y = 0$ (x-axis).

$y = x^2 - 8x + 15$ ← graph
 $y = 0$ ← x-axis (System of equations)

$y = x^2 - 8x + 15$



$$y = x^2 - 8x + 15 \quad \text{Use substitution method to solve.}$$
$$y = 0$$

$$x^2 - 8x + 15 = 0$$
$$(x-3)(x-5) = 0$$

Solve for x .

Use the zero product property to solve for x .

$$x - 3 = 0$$
$$+3 = +3$$

$$x = 3$$

$$x - 5 = 0$$
$$+5 = 5$$

$$x = 5$$

The x -intercepts are $(3,0)$ and $(5,0)$.

Zero product Property

If $a \cdot b = 0$, then $a = 0$ or $b = 0$ (or both $a = 0$ and $b = 0$).

Conclusion One way to solve quadratics is to:

① set quadratic equation equal to 0

② factor quadratic

③ use zero product property.

[Examples] Solve for x .

Case A Solving Quadratics — Factoring Trinomials (set equal to 0)

$$\textcircled{1} x^2 + 3x - 18 = 0$$
$$(x+6)(x-3) = 0$$
$$x+6 = 0 \quad \text{or} \quad x-3 = 0$$
$$x = -6 \quad \quad \quad x = 3$$

$$\textcircled{2} x^2 + 10x + 16 = 0$$
$$(x+2)(x+8) = 0$$
$$x+2 = 0 \quad \text{or} \quad x+8 = 0$$
$$x = -2 \quad \quad \quad x = -8$$

Case B Solve Quadratics — Difference of Squares (set equal to 0)

$$\begin{aligned} \textcircled{3} \quad x^2 - 49 &= 0 \\ x^2 + 0x - 49 &= 0 \quad \begin{array}{l} \text{1} \cdot \text{49} \\ \text{-7} \cdot \text{-7} \end{array} \\ (x+7)(x-7) &= 0 \\ x+7=0 \text{ or } x-7=0 \\ x=-7 \text{ or } x=7 \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad x^2 - 36 &= 0 \\ (x+6)(x-6) &= 0 \\ x+6=0 \text{ or } x-6=0 \\ x=-6 \text{ or } x=6. \end{aligned}$$

Case C Solve Quadratics — GCF Factoring (set equal to 0)

$$\begin{aligned} \textcircled{5} \quad x^2 + 4x &= 0 \\ x(x+4) &= 0 \\ x=0 \text{ or } x+4=0 \\ x=0 \text{ or } x=-4. \end{aligned}$$

$$\begin{aligned} \textcircled{6} \quad 5x^2 + 10x &= 0 \\ 5x(x+2) &= 0 \\ 5x=0 \text{ or } x+2=0 \\ x=0 \text{ or } x=-2. \end{aligned}$$

$$\begin{aligned} \textcircled{7} \quad 4x^2 - 16x &= 0 \\ 4x(x-4) &= 0 \\ 4x=0 \text{ or } x-4=0 \\ x=0 \text{ or } x=4. \end{aligned}$$

$$\begin{aligned} \textcircled{8} \quad 2x^2 - 14x + 12 &= 0 \\ 2(x^2 - 7x + 6) &= 0 \\ 2(x-1)(x-6) &= 0 \\ x-1=0 \text{ or } x-6=0 \\ x=1 \text{ or } x=6. \end{aligned}$$

[More Examples] Solve for x.

$$\begin{aligned} \textcircled{9} \quad 2x^2 - 5x + 2 &= 0 \\ (2x-1)(x-2) &= 0 \\ 2x-1=0 \text{ or } x-2=0 \\ \begin{array}{r} 2x-1=0 \\ +1 \quad +1 \\ \hline 2x=1 \\ \frac{2x}{2} = \frac{1}{2} \\ x = \frac{1}{2} \end{array} \quad x=2 \end{aligned}$$

$$\begin{aligned} \textcircled{10} \quad x^2 - 6x &= 9 \quad \text{Set equal to 0.} \\ \quad \quad \quad -9 &= -9 \\ \hline x^2 - 6x - 9 &= 0 \quad \text{Solve for x.} \\ (x-3)(x-3) &= 0 \\ x-3=0 \text{ or } x-3=0 \\ x=3 \end{aligned}$$

$$\textcircled{11} \quad \begin{array}{r} x^2 - 3 = -2x \\ +2x \quad +2x \\ \hline x^2 + 2x - 3 = 0 \\ (x+3)(x-1) = 0 \\ x+3=0 \quad \text{or} \quad x-1=0 \\ x=-3 \quad \quad \quad x=1. \end{array}$$

$$\textcircled{12} \quad \begin{array}{r} 3x^2 = -9x \\ +9x = +9x \\ \hline 3x^2 + 9x = 0 \\ 3x(x+3) = 0 \\ 3x=0 \quad \text{or} \quad x+3=0 \\ x=0 \quad \text{or} \quad x=-3 \end{array}$$

$$\textcircled{13} \quad \begin{array}{r} 3x^2 = 27 \\ -27 = -27 \\ \hline 3x^2 - 27 = 0 \\ 3(x^2 - 9) = 0 \\ 3(x+3)(x-3) = 0 \\ x+3=0 \quad \text{or} \quad x-3=0 \\ x=-3 \quad \text{or} \quad x=3. \end{array}$$

WAYS TO SOLVE QUADRATICS

(Quad & Linear or Constant) 2 Terms

- GCF Factoring (set = 0)
- Difference of Squares (set = 0)

3 Terms (Quad, Linear, Constant)

- GCF Factoring (set = 0)
- Factoring Trinomials (set = 0)