

1.7 Factoring Quadratics

Part 1

GCF Factoring, Difference of Squares, & Factoring
Trinomials ($a=1$)

Old Multiplying Polynomials

Expand the product.

Distribution Property or First Outer Inner Last

① $(x+3)(x-4)$

$$= x^2 - 4x + 3x - 12$$
$$= x^2 - x - 12$$

② $(x-5)(x+5)$

$$= x^2 + 5x - 5x - 25$$
$$= x^2 - 25$$

③ $x(7x+6)$

$$= 7x^2 + 6x$$

④ $4x(3+7x^2)$

$$= 12x + 28x^3$$

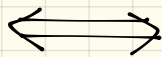
New Factoring Part 1

Case 1 GCF Factoring - finding the **greatest common factor (GCF)** of a sum of terms. Use the **GCF** to create a product.

Let's consider the expression $2x^2(7x+3)$. Expand the expression.

~~Before~~ **FORWARDS** (Expand)

$$2x^2(7x+3)$$
$$= 14x^3 + 6x^2$$



~~Now~~ **BACKWARDS** (Factor)

$$14x^3 + 6x^2$$
$$2x^2(7x+3)$$

GCF for #'s
14 6
1·14 1·6
② 7 ② 3

GCF for variable

x^3 x^2
 $x \cdot x \cdot x$ $x \cdot x$

[More Examples] Factor the problems.

$$\textcircled{1} 5x^2 + 15x$$

$$5x(1x + 3)$$

GCF for #'s

15	5
1 · 15	1 · 5
3 · 5	

GCF for variable

x^2	x
$\textcircled{x} \cdot x$	\textcircled{x}

$$\textcircled{2} 2x^3 - 8x^2$$

$$2x^2(x - 8)$$

GCF for #'s

2	8
1 · 2	1 · 8
	2 · 4

GCF for variable

x^3	x^2
$\textcircled{x} \cdot \textcircled{x} \cdot x$	$\textcircled{x} \cdot \textcircled{x}$

$$\textcircled{3} 2x^2 - 4x$$

$$2x(x - 2)$$

GCF for #'s

2	4
1 · 2	1 · 4
	2 · 2

GCF for variable

x^2	x
$\textcircled{x} \cdot x$	\textcircled{x}

$$\textcircled{4} 15x^2 - 5x + 30$$

$$5(3x^2 - x + 6)$$

GCF for #'s

15	5	30
1 · 15	1 · 5	1 · 30
3 · 5		2 · 15
		3 · 10
		5 · 6

GCF for variable

x^2	x	1
$x \cdot x$	x	1

Case B Factoring Trinomials — factor the sum of 3 terms (trinomial) into a product of 2 binomials.

Let's consider the expression $(x+3)(x+2)$. Expand the expression.

$$\textcircled{(x+3)}\textcircled{(x+2)} = x^2 + 2x + 3x + 6$$

$$= x^2 + 5x + 6$$

Standard Form for Trinomials $\Rightarrow ax^2 + bx + c$

$$ax^2 + \underbrace{bx + c}_{\text{sum}} \rightarrow \text{product}$$

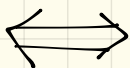
Guess & Check Method

To factor the trinomial: Find 2 numbers that multiplies to get the "product" (c) & adds up to get the "sum" (b).

~~Before~~ (Expand)
FORWARDS

$$(x+3)(x+2)$$

$$= x^2 + 2x + 3x + 6$$
$$= x^2 + 5x + 6$$



~~Now~~ (Factor)
BACKWARDS

$$x^2 + 5x + 6$$

Factors of "c"
 $\begin{matrix} 1 \cdot 6 \\ 2 \cdot 3 \end{matrix}$ - multiplies to 6 & adds to 5.

$$= (x+2)(x+3)$$

[Examples] Factor the trinomial.

$$\textcircled{1} x^2 + 9x + 14 \begin{matrix} 1 \cdot 14 \\ 2 \cdot 7 \end{matrix}$$
$$= (x+2)(x+7)$$

$$\textcircled{2} n^2 - 11n + 10 \begin{matrix} 1 \cdot 10 \\ 2 \cdot 5 \end{matrix}$$
$$= (n-1)(n-10)$$

$$\textcircled{3} n^2 + 4n - 12 \begin{matrix} 1 \cdot 12 \\ 2 \cdot 6 \\ 4 \cdot 3 \end{matrix}$$
$$= (n-2)(n+6)$$

$$\textcircled{4} m^2 + 2m - 24 \begin{matrix} 1 \cdot 24 \\ 2 \cdot 12 \\ 3 \cdot 8 \\ 4 \cdot 6 \end{matrix}$$
$$= (m-4)(m+6)$$

$$\textcircled{5} k^2 - 13k + 40 \begin{matrix} 1 \cdot 40 \\ 2 \cdot 20 \\ 4 \cdot 10 \\ 5 \cdot 8 \end{matrix}$$
$$= (k-5)(k-8)$$

$$\textcircled{6} 4v^2 - 4v - 8 \begin{matrix} 1 \cdot 2 \end{matrix}$$
$$= 4(v^2 - v - 2)$$
$$= 4(v-2)(v+1)$$

Case C Difference of Squares

Let's consider the expression $(x-5)(x+5)$. Expand the expression.

$$\begin{aligned}(x+5)(x-5) &= x^2 - 5x + 5x - 25 \\ &= x^2 - 25\end{aligned}$$

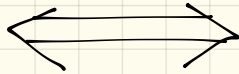
the middle term (linear term) was eliminated because "5x" and "-5x" are additive inverses.

Difference of Squares Mean:

"perfect square" minus "perfect square" $a^2 - b^2 = (\sqrt{a^2} - \sqrt{b^2})(\sqrt{a^2} + \sqrt{b^2})$

Before FORWARD (Expand)

$$\begin{aligned}(x+5)(x-5) \\ = x^2 - 5x + 5x - 25 \\ = x^2 - 25\end{aligned}$$



Now BACKWARD (Factor)

$$\begin{aligned}x^2 - 25 \\ = x^2 + 0x - 25 \quad \frac{1}{5} \cdot 25 \\ = (x-5)(x+5)\end{aligned}$$

[Examples]

① $x^2 - 36$
 $(x+6)(x-6)$

② $x^2 - 4$
 $(x+2)(x-2)$

③ $4w^2 - 16$
 $4(w^2 - 4)$
 $4(w+2)(w-2)$