1.7 Factoring Quadratics

Part 1

GCF Factoring, Difference of Squares, & Factoring Trinomials $(a=1)$

$\boxed{\text{Old}}$ Multiplying Polynomials

Distribution Property or First Outer Inner Last

Expand the product.

① $(x+3)(x-4)$

$= x^2 - 4x + 3x - 12$

$= x^2 - x - 12$

② $(x-5)(x+5)$

$= x^2 + 5x - 5x - 25$

$= x^2 - 25$

③ $x(7x+6)$

$= 7x^2 + 6x$

④ $4x(3+7x^2)$

$= 12x + 28x^3$

$\boxed{\text{now}}$ Factoring Part 1

Case 1 GCF Factoring – finding the greatest common factor (GCF) of a sum of terms. Use the GCF to create a product.

Let's consider the expression $2x^2(7x+3)$. Expand the expression.

Before FORWARDS (Expand) ⟺ Now BACKWARDS (Factor)

$2x^2(7x+3)$

$= 14x^3 + 6x^2$

$14x^3 + 6x^2$

$2x^2(7x+3)$

GCF for #'s

14	6
$1 \cdot 14$	$1 \cdot 6$
$2 \cdot 7$	$2 \cdot 3$

GCF for variable

x^3	x^2
$x \cdot x \cdot x$	$x \cdot x$

[More Examples] Factor the problems.

① $5x^2 + 15x$

$5x(1x + 3)$

GCF for #'s

15: $1 \cdot 15$, $3 \cdot 5$

5: $1 \cdot 5$

GCF for variable

x^2: $x \cdot x$

x: x

② $2x^3 - 8x^2$

$2x^2(x - 8)$

GCF for #'s

2: $1 \cdot 2$

8: $1 \cdot 8$, $2 \cdot 4$

GCF for variable

x^3: $x \cdot x \cdot x$

x^2: $x \cdot x$

③ $2x^2 - 4x$

$2x(x - 2)$

GCF for #'s

2: $1 \cdot 2$

4: $1 \cdot 4$, $2 \cdot 2$

GCF for variable

x^2: $x \cdot x$

x: x

④ $15x^2 - 5x + 30$

$5(3x^2 - x - 6)$

GCF for #'s

15: $1 \cdot 15$, $3 \cdot 5$

5: $1 \cdot 5$

30: $1 \cdot 30$, $2 \cdot 15$, $3 \cdot 10$, $5 \cdot 6$

GCF for variable

x^2: $x \cdot x$

x: x

1: 1

Case B <u>Factoring Trinomials</u> – factor the sum of 3 terms (trinomial) into a product of 2 binomials.

Let's consider the expression $(x+3)(x+2)$. Expand the expression.

$$(x+3)(x+2) = x^2 + 2x + 3x + 6$$
$$= x^2 + 5x + 6$$

Standard Form for Trinomials $\Longrightarrow ax^2+bx+c$

$$ax^2+\underbrace{bx}_{\text{sum}}+\underbrace{c}_{\text{product}}$$

$\boxed{\text{Guess \& Check Method}}$

To factor the trinomial: Find 2 numbers that multiplies to get the "product" (c) & adds up to get the "sum" (b).

Before FORWARDS (Expand) $\Longleftrightarrow$ Now BACKWARDS (Factor)

FORWARDS (Expand)

$(x+3)(x+2)$

$= x^2+2x+3x+6$

$= x^2+5x+6$

BACKWARDS (Factor)

x^2+5x+6

Factors of "c"

$1\cdot 6$

$2\cdot 3$

multiplies to 6 & adds to 5.

$= (x+2)(x+3)$

[Examples] Factor the trinomial.

① $x^2+9x+14$ — $1\cdot 14$, $2\cdot 7$

$=(x+2)(x+7)$

② $n^2-11n+10$ — $-1\cdot -10$, $2\cdot 5$

$=(n-1)(n-10)$

③ $n^2+4n-12$ — $1\cdot 12$, $-2\cdot 6$, $4\cdot 3$

$=(n-2)(n+6)$

④ $m^2+2m-24$ — $1\cdot 24$, $2\cdot 12$, $3\cdot 8$, $-4\cdot 6$

$=(m-4)(m+6)$

⑤ $k^2-13k+40$ — $1\cdot 40$, $2\cdot 20$, $4\cdot 10$, $-5\cdot -8$

$=(k-5)(k-8)$

⑥ $4v^2-4v-8$

$=4(v^2-v-2)$ — $1\cdot 2$

$=4(v-2)(v+1)$

Case C Difference of Squares

Let's consider the expression $(x-5)(x+5)$. Expand the expression.

$(x+5)(x-5) = x^2-5x+5x-25$
$= x^2-25$ ← the middle term (linear term) was eliminated because "5x" and "-5x" are additive inverses.

Difference of Squares Mean:
"perfect square" minus "perfect square" $a^2-b^2=(\sqrt{a^2}-\sqrt{b^2})(\sqrt{a^2}+\sqrt{b^2})$

Before FORWARD (Expand) ⟺ Now BACKWARD (Factor)

FORWARD:
$(x+5)(x-5)$
$= x^2-5x+5x-25$
$= x^2-25$

BACKWARD:
x^2-25
$= x^2+0x-25$ $\quad 1 \cdot 25$, $-5 \cdot 5$
$= (x-5)(x+5)$

[Examples]

① x^2-36
$(x+6)(x-6)$

② x^2-4
$(x+2)(x-2)$

③ $4w^2-16$
$4(w^2-4)$
$4(w+2)(w-2)$