

## 1.4 Rational Exponents

# Old Simplify

$$\textcircled{1} i^{106} = -1$$

$$\frac{106}{4} = 26.5$$

$$\begin{aligned}
 i &= \sqrt{-1} \rightarrow .25 \\
 i^2 &= -1 \rightarrow .50 \\
 i^3 &= -i \rightarrow .75 \\
 i^4 &= 1 \rightarrow 0
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} (3+i)(4-2i) &= 12-6i+4i-2i^2 \\
 &= 12-2i-2(-1) \\
 &= 12+2-2i \\
 &= 14-2i
 \end{aligned}$$

First  
Outer  
Inner  
Last

$$\begin{aligned}
 \textcircled{3} \sqrt{-80} &= \sqrt{80} \cdot \sqrt{-1} \\
 &= \sqrt{16} \cdot \sqrt{5} \cdot i \\
 &= 4i\sqrt{5}
 \end{aligned}$$

1  
4  
9  
16  
25  
36  
49  
:

## Reducing Radicals involving Variables

$$\begin{aligned}
 \textcircled{1} \sqrt{x^2} &= \sqrt{x} \cdot \sqrt{x} \\
 &= x
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} \sqrt{x^4} &= \sqrt{x^2} \cdot \sqrt{x^2} \\
 &= x^2
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{3} \sqrt{x^6} &= \sqrt{x^3} \cdot \sqrt{x^3} \\
 &= x^3
 \end{aligned}$$

conclusion: even number exponents  $\Rightarrow$  divide by 2

$$\begin{aligned}
 \textcircled{1} \sqrt{x^3} &= \sqrt{x^2} \cdot \sqrt{x} \\
 &= x\sqrt{x}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} \sqrt{x^5} &= \sqrt{x^4} \cdot \sqrt{x} \\
 &= x^2\sqrt{x}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{3} \sqrt{x^{11}} &= \sqrt{x^{10}} \cdot \sqrt{x} \\
 &= x^5\sqrt{x}
 \end{aligned}$$

conclusion: odd number exponents  $\Rightarrow$  subtract the exponent by 1 & simplify the even exponent's radical.

## [More Examples]

$$\begin{aligned} \textcircled{1} \sqrt{45x^2} &= \sqrt{9} \cdot \sqrt{5} \cdot \sqrt{x^2} \\ &= 3 \cdot \sqrt{5} \cdot x \\ &= 3x\sqrt{5} \end{aligned}$$

$$\begin{aligned} \textcircled{2} \sqrt{200x^5} &= \sqrt{100} \cdot \sqrt{2} \cdot \sqrt{x^4} \cdot \sqrt{x} \\ &= 10 \cdot \sqrt{2} \cdot x^2 \cdot \sqrt{x} \\ &= 10x^2\sqrt{2x} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \sqrt{80x^{13}} &= \sqrt{16} \cdot \sqrt{5} \cdot \sqrt{x^{12}} \cdot \sqrt{x} \\ &= 4 \cdot \sqrt{5} \cdot x^6 \cdot \sqrt{x} \\ &= 4x^6\sqrt{5x} \end{aligned}$$

## [NEW] Rewriting $N^{\text{th}}$ Roots & Radical Exponents

Parts of a Radical:  $\text{root}\sqrt{\text{radicand}}$

(Examples)

$$\sqrt{2}, \sqrt[3]{2}, \sqrt[4]{2}, \sqrt[5]{2}, \dots$$

↑  
Square  
root

↑  
Cubic  
root

↑  
fourth  
root

↑  
fifth  
root

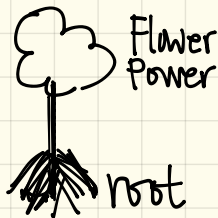
(no number  
where the root  
is one is a  
square root)

## Rewriting Radicals to Rational Exponents

$$(\text{root}\sqrt{\text{radicand}})^{\text{power}} = \text{radicand}^{\frac{\text{power}}{\text{root}}}$$

"Power over root"

"Power is on top, Roots are on the ground."



[Examples] Rewrite with a Rational Exponent.

Rewriting Radicals to Rational Exponents

$$\textcircled{1} \sqrt{10w} = (\sqrt{10w})^1 = (10w)^{\frac{1}{2}}$$

$$(\text{root/radical})^{\text{power}} = \text{radical}^{\frac{\text{power}}{\text{root}}}$$

$$\textcircled{2} \sqrt[3]{7p} = (\sqrt[3]{7p})^1 = (7p)^{\frac{1}{3}}$$

$$\textcircled{3} \sqrt{y^2} = y^{\frac{2}{2}} = y^1$$

$$\textcircled{4} \sqrt[3]{z^6} = z^{\frac{6}{3}} = z^2$$

[Examples] Rewrite Rational Exponents to Radicals

Rewriting Rational Exponents to Radicals

$$\textcircled{1} 12^{\frac{3}{5}} = \sqrt[5]{12^3}$$

$$\textcircled{2} 13^{\frac{2}{3}} = \sqrt[3]{13^2}$$

$$\textcircled{3} x^{\frac{2}{3}} = \sqrt[3]{x^2}$$

$$\textcircled{4} 17^{\frac{5}{2}} = \sqrt{17^5}$$

$$\textcircled{5} (7p)^{\frac{1}{3}} = \sqrt[3]{7p}$$

$$\text{radical}^{\frac{\text{power}}{\text{root}}} = (\text{root/radical})^{\text{power}}$$