1.2 Imaginary Numbers

$\boxed{\text{Old}}$ Simplifying Radicals

Perfect Square List

1
4
9
16
25
36
49
64
81
100
⋮

① $\sqrt{20}$
$= \sqrt{4} \cdot \sqrt{5}$
$= 2\sqrt{5}$

② $\sqrt{200}$
$= \sqrt{100} \cdot \sqrt{2}$
$= 10\sqrt{2}$

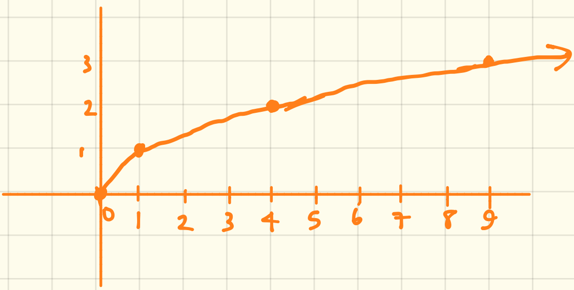
③ $3\sqrt{96}$
$= 3 \cdot \sqrt{16} \cdot \sqrt{6}$
$= 3 \cdot 4 \cdot \sqrt{6}$
$= 12\sqrt{6}$

*Try inputting #1-3 radicals inside your Multiview calculator.
(You should get the same reduced radicals)

Try inputting $\sqrt{-20}$ into your Multiview calculator.
(You should get a domain error statement).

Why can't negative numbers be under radicals?

Let's consider $f(x) = \sqrt{x}$. Create a table & graphical representations for this function.



x	y
0	0
1	1
2	~1.42
3	~1.73
4	2
5	~2.24
⋮	⋮

Conclusion: The DOMAIN of $f(x)$ does not include NEGATIVE NUMBERS.

$\boxed{\text{new}}$ Imaginary Numbers

$\boxed{\text{FACT:}}$ $\sqrt{-1} = i$ $\Rightarrow$ In the "imaginary world", you can have a negative under the radical. You will bring out an "i" (imaginary).

① $\sqrt{-16}$
$= \sqrt{16} \cdot \sqrt{-1}$
$= 4i$

② $\sqrt{-81}$
$= \sqrt{81} \cdot \sqrt{-1}$
$= 9i$

③ $\sqrt{-45}$
$= \sqrt{45} \cdot \sqrt{-1}$
$= \sqrt{9} \cdot \sqrt{5} \cdot \sqrt{-1}$
$= 3i\sqrt{5}$

④ $\sqrt{-200}$
$= \sqrt{200} \cdot \sqrt{-1}$
$= \sqrt{100} \cdot \sqrt{2} \cdot \sqrt{-1}$
$= 10i\sqrt{2}$

$\boxed{\text{Powers of } i}$ Let's recall that $\sqrt{-1} = i$

$i = \sqrt{-1}$

$i^2 = \sqrt{-1} \cdot \sqrt{-1}$
$= -1$

$i^3 = \sqrt{-1} \cdot \sqrt{-1} \cdot \sqrt{-1}$
$= -1\sqrt{-1}$
$= -\sqrt{-1}$
$= -i$

$i^4 = \sqrt{-1} \cdot \sqrt{-1} \cdot \sqrt{-1} \cdot \sqrt{-1}$
$= -1 \cdot -1$
$= 1$

$i = \sqrt{-1}$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$

$i^5 = i^4 \cdot i$
$= 1 \cdot \sqrt{-1}$
$= \sqrt{-1}$

$i^6 = i^5 \cdot i$
$= \sqrt{-1} \cdot \sqrt{-1}$
$= -1$

$i^7 = i^6 \cdot i$
$= -1 \cdot \sqrt{-1}$
$= -\sqrt{1}$

$i^8 = i^7 \cdot i$
$= -\sqrt{-1} \cdot \sqrt{-1}$
$= -(-1)$
$= 1$

$i = \sqrt{-1}$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$

$i^5 = \sqrt{-1}$
$i^6 = -1$
$i^7 = -i$
$i^8 = 1$

$i^9 = \sqrt{-1}$
$i^{10} = -1$
$i^{11} = -i$
$i^{12} = 1$

What if I asked for i^{75}?

Use calculator!

$\frac{75}{4} = 18.75$

$i = \sqrt{-1} \rightarrow .25$
$i^2 = -1 \rightarrow .50$
$i^3 = -i \rightarrow .75$
$i^4 = 1 \rightarrow 0$

So, $i^{75} = -i$.

[More Examples]

① $i^{29} = i$

$\frac{29}{4} = 7.25$

③ $i^{9536} = 1$

$\frac{9536}{4} = 2384.0$

② $i^{251} = i^3$

$\frac{251}{4} = 62.75$

$\boxed{\text{Complex Number}}$

Standard Form $\Rightarrow \underbrace{a}_{\text{Real Number}} + \underbrace{bi}_{\text{Imaginary Number}}$

Operations involving Complex Numbers

1. Treat i like variables
2. Combine Like Terms
3. Simplify (no powers of i higher than 1 are allowed)
4. All answers need to be in Standard Form a+bi

Add & Subtracting Complex Numbers

1) $(3+2i)+(7+6i)$
$= 3+2i+7+6i$
$= 3+7+2i+6i$
$= 10+8i$

2) $(6+5i)-(-1+2i)$
$= 6+5i+1-2i$
$= 6+1+5i-2i$
$= 7-3i$

3) $(9-4i)-(2-3i)$
$= 9-4i-2+3i$
$= 9-2-4i+3i$
$= 7-i$

4) $9-(10+2i)-5i$
$= 9-10-2i-5i$
$= -1-7i$

5) $(11i^4+4i^3)-(2i^4-6i^3)$
$= 11i^4+4i^3-2i^4+6i^3$
$= 11i^4-2i^4+4i^3+6i^3$
$= 9i^4+10i^3$
$= 9(1)+10(-i)$
$= 9-10i$

$i=\sqrt{-1}$
$i^2=-1$
$i^3=-i$
$i^4=1$

Multiplying Complex Numbers

Remember Distributive Property & FOIL

First
Outer
Inner
Last

1) $-i(3+i)$
$= -3i-i^2$
$= -3i-(-1)$
$= -3i+1$
$= 1-3i$

2) $(2+3i)(-6-2i)$
$= -12-4i-18i+6i^2$
$= -12-22i+6i^2$
$= -12-22i+6(-1)$
$= -12-22i+6$
$= -12+6-22i$
$= -6-22i$

$i=\sqrt{-1}$
$i^2=-1$
$i^3=-i$
$i^4=1$

③ $(-3+i)(8+5i)$

$= -24 - 15i + 8i + 5i^2$
$= -24 - 7i + 5(-1)$
$= -24 - 7i - 5$
$= -24 - 5 - 7i$
$= -19 - 7i$

④ $-2i(1+4i)$
$= -2i + 8i^2$
$= -2i + 8(-1)$
$= -2i - 8$

Dividing Complex Numbers

No radicals are allowed in the denominator! Use the conjugate to get in standard form.

① $\frac{4}{i} \cdot \frac{i}{i} = \frac{4i}{i^2} = \frac{4i}{(-1)} = -4i$

② $\frac{10+i}{i} \cdot \frac{i}{i} = \frac{10i+i^2}{i^2} = \frac{10i+(-1)}{(-1)} = \frac{10i-1}{-1} = -10i+1 = 1-10i$

③ $\frac{2+3i}{5i} \cdot \frac{5i}{5i} = \frac{10i+15i^2}{25i^2} = \frac{10i+15(-1)}{25(-1)} = \frac{10i-15}{25} = \frac{10i}{25} - \frac{15}{25}$

$= \frac{2}{5}i - \frac{3}{5}$

Let's consider the standard form of Complex Numbers: $a+bi$.

The conjugate of $a+bi$ is $a-bi$.

Basically: Use the opposite sign in between the 2 terms

④ $\frac{10}{2-4i} \cdot \frac{2+4i}{2+4i} = \frac{20+40i}{4+8i-8i-16i^2} = \frac{20+40i}{4-16i^2} = \frac{20+40i}{4-16(-1)}$

$= \frac{20+40i}{4+16} = \frac{20+40i}{20} = \frac{20}{20} + \frac{40i}{20} = 1+2i$

⑤ $\frac{2+3i}{1+2i} \cdot \frac{1-2i}{1-2i} = \frac{2-4i+3i-6i^2}{1-2i+2i-4i^2} = \frac{2-i-6(-1)}{1-4(-1)}$

$= \frac{2+6-i}{1+4} = \frac{8-i}{5}$