

1.14 Changing Forms

Converting between Vertex & Standard Forms

Old Quadratics Transformations

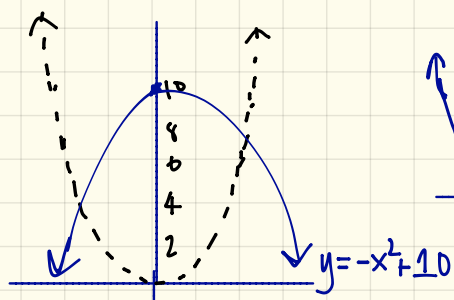
Let's recall the Standard & Vertex Forms of Quadratics.

Standard Form
 $y = ax^2 + bx + c$

Vertex Form
 $y = a(x-h)^2 + k$

① $f(x) = -x^2 + 10$

shift down 10, ↻



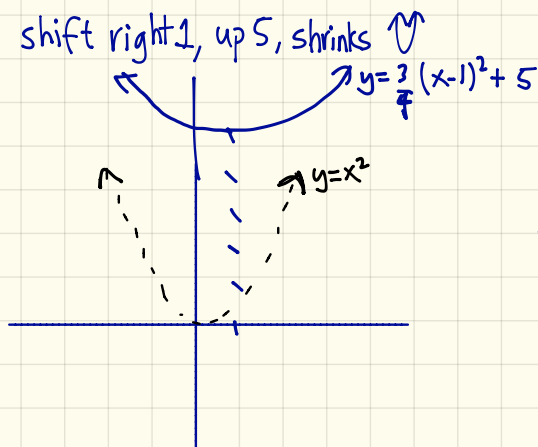
② $f(x) = (x+5)^2$

shift left 5, ↻



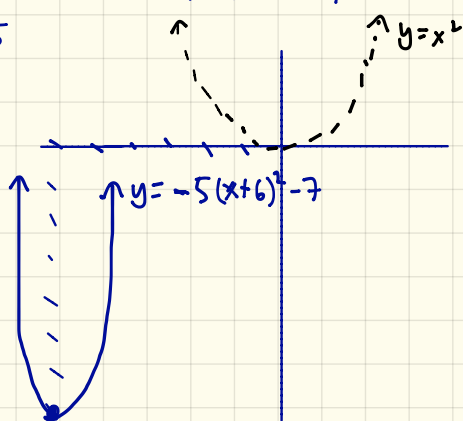
③ $f(x) = \frac{3}{7}(x-1)^2 + 5$

shift right 1, up 5, shrinks ↻



④ $f(x) = -5(x+6)^2 - 7$

shift left 6, down 7, stretch ↻



New Changing Forms

① Standard to Vertex \longrightarrow Use the Completing the Square Method

- Set the problem up to complete the square $\Rightarrow y = ax^2 + bx + c$
(Factor out a number if needed.)
- Add & Subtract the square of half of the coefficient of the linear term.
- Combine like terms
- Write the perfect square trinomial as a binomial squared.
 $\Rightarrow y = a(x-h)^2 + k$

[Examples] Convert Standard to Vertex

$$\begin{aligned} \textcircled{1} \quad y &= x^2 + 8x + 10 \\ y &= (x^2 + 8x) + 10 \\ &= (x^2 + 8x + \underline{\quad}) + 10 - \underline{\quad} \\ &= (x^2 + 8x + (\frac{8}{2})^2) + 10 - (\frac{8}{2})^2 \\ &= (x^2 + 8x + 16) + 10 - 16 \\ &= (x+4)(x+4) - 6 \\ &= (x+4)^2 - 6 \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad y &= x^2 + 6x + 8 \\ y &= (x^2 + 6x) + 8 \\ &= (x^2 + 6x + \underline{\quad}) + 8 - \underline{\quad} \\ &= (x^2 + 6x + (\frac{6}{2})^2) + 8 - (\frac{6}{2})^2 \\ &= (x^2 + 6x + 9) + 8 - 9 \\ &= (x+3)(x+3) - 1 \\ &= (x+3)^2 - 1 \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad f(x) &= x^2 - 4x + 3 \\ &= (x^2 - 4x) + 3 \\ &= (x^2 - 4x + \underline{\quad}) + 3 - \underline{\quad} \\ &= (x^2 - 4x + (\frac{4}{2})^2) + 3 - (\frac{4}{2})^2 \\ &= (x^2 - 4x + 4) + 3 - 4 \\ &= (x-2)(x-2) - 1 \\ &= (x-2)^2 - 1 \end{aligned}$$

$$\begin{aligned} \textcircled{4} \quad y &= 3x^2 + 24x + 50 \\ &= (3x^2 + 24x) + 50 \\ &= 3(x^2 + 8x) + 50 \\ &= 3(x^2 + 8x + \underline{\quad}) + 50 - 3(\underline{\quad}) \\ &= 3(x^2 + 8x + (\frac{8}{2})^2) + 50 - 3(\frac{8}{2})^2 \\ &= 3(x^2 + 8x + 16) + 50 - 48 \\ &= 3(x+4)(x+4) + 2 \\ &= 3(x+4)^2 + 2 \end{aligned}$$

Another way to put in vertex form. \rightarrow use vertex formula

First Vertex Formula: $x = \frac{-b}{2a}$, $y = f\left(\frac{-b}{2a}\right)$.

Lastly Vertex Form: $y = a(x-h)^2 + k$

[Examples] Convert Standard to Vertex

① $y = x^2 + 8x + 10$ $\begin{matrix} a=1 \\ b=-8 \\ c=10 \end{matrix}$

$$x = \frac{-b}{2a} = \frac{-(-8)}{2(1)} = -4$$

$$y = f(-4) = (-4)^2 + 8(-4) + 10 = -6$$

$$\begin{aligned} y &= a(x-h)^2 + k \\ &= (x - (-4))^2 - 6 \\ &= (x + 4)^2 - 6 \end{aligned}$$

② $y = x^2 + 6x + 8$ $\begin{matrix} a=1 \\ b=6 \\ c=8 \end{matrix}$

$$x = \frac{-b}{2a} = \frac{-(6)}{2(1)} = -3$$

$$y = f(-3) = (-3)^2 + 6(-3) + 8 = -1$$

$$\begin{aligned} y &= a(x-h)^2 + k \\ &= (x - (-3))^2 - 1 \\ &= (x + 3)^2 - 1 \end{aligned}$$

③ $f(x) = x^2 - 4x + 3$ $\begin{matrix} a=1 \\ b=-4 \\ c=3 \end{matrix}$

$$x = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = 2$$

$$y = f(2) = (2)^2 - 4(2) + 3 = -1$$

$$\begin{aligned} y &= a(x-h)^2 + k \\ &= (x - 2)^2 - 1 \end{aligned}$$

④ $y = 3x^2 + 24x + 50$ $\begin{matrix} a=3 \\ b=24 \\ c=50 \end{matrix}$

$$x = \frac{-b}{2a} = \frac{-(24)}{2(3)} = -4$$

$$y = f(-4) = 3(-4)^2 + 24(-4) + 50 = 2$$

$$\begin{aligned} y &= a(x-h)^2 + k \\ &= 3(x - (-4))^2 + 2 \\ &= 3(x + 4)^2 + 2 \end{aligned}$$

② Vertex to Standard \longrightarrow Expand the equation.

· Expand the equation by multiplying & combining like terms.

[Examples] Convert Vertex to Standard.

$$\begin{aligned}\textcircled{1} \quad y &= (x-1)^2 + 8 \\ &= (x-1)(x-1) + 8 \\ &= x^2 - 1x - 1x + 1 + 8 \\ &= x^2 - 2x + 9\end{aligned}$$

$$\begin{aligned}\textcircled{2} \quad y &= -(x-4)^2 + 3 \\ &= -(x-4)(x-4) + 3 \\ &= -(x^2 - 4x - 4x + 16) + 3 \\ &= -(x^2 - 8x + 16) + 3 \\ &= -x^2 + 8x - 16 + 3 \\ &= -x^2 + 8x - 13\end{aligned}$$

$$\begin{aligned}\textcircled{3} \quad f(x) &= 2(x+3)^2 - 5 \\ &= 2(x+3)(x+3) - 5 \\ &= 2(x^2 + 3x + 3x + 9) - 5 \\ &= 2(x^2 + 6x + 9) - 5 \\ &= 2x^2 + 12x + 18 - 5 \\ &= 2x^2 + 12x + 13\end{aligned}$$

$$\begin{aligned}\textcircled{4} \quad f(x) &= 2(x+1)^2 - 2 \\ &= 2(x+1)(x+1) - 2 \\ &= 2(x^2 + 1x + 1x + 1) - 2 \\ &= 2(x^2 + 2x + 1) - 2 \\ &= 2x^2 + 4x + 2 - 2 \\ &= 2x^2 + 4x\end{aligned}$$