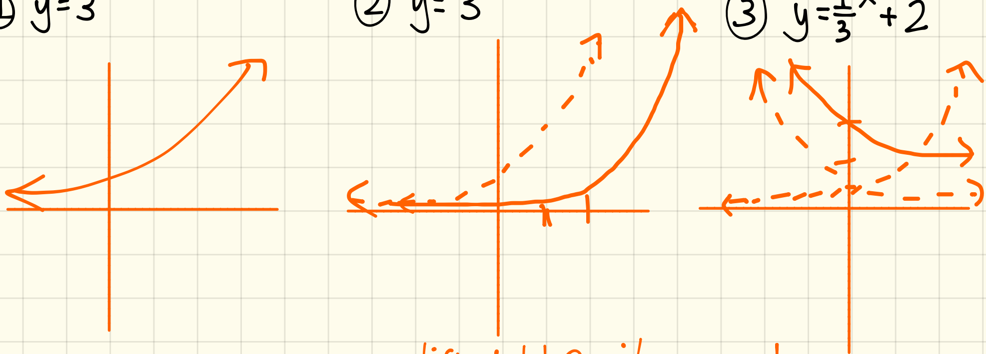
1.12 Graph Vertex Form & Transformations of Quadratics

$\boxed{\text{old}}$ Exponential Transformations

① $y=3^x$ ② $y=3^{x-2}$ ③ $y=\frac{1}{3}^x+2$



- shift right 2 units

- decay
- shift up 2 units

The parent function of exponential function is $f(x)=a(b)^x$.

Recall Rules

· up	$y=a(r)^x+k$	· left	$y=a(r)^{x+k}$	· growth	$a>1$
· down	$y=a(r)^x-k$	· right	$y=a(r)^{x-k}$	· decay	$0<a<1$

more old...

Recall standard form for Quadratics $\Rightarrow ax^2+bx+c$.

Solve the Quadratic equation & sketch the meaning graphically.

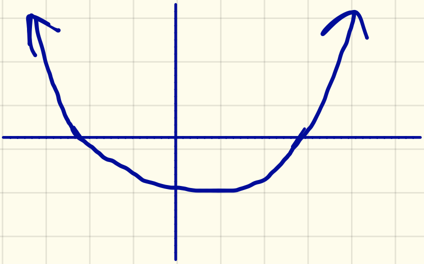
$x^2-x=6$

$x^2-x-6=0$ 1·6, -3·2

$(x-3)(x+2)=0$

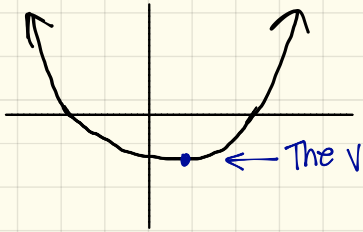
$x-3=0$ or $x+2=0$

$x=3$ $x=-2$.

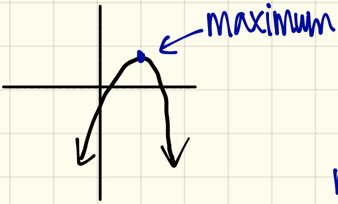


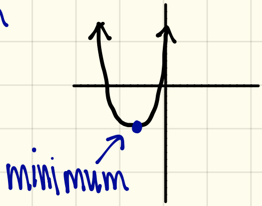
$\boxed{\text{new}}$ Transformations of Quadratics in vertex form

Let's consider the graph of the previous example.



A vertex gives the <u>maximum</u> or <u>minimum</u> value of a Quadratic.





Vertex Form of Quadratics $\Longrightarrow$ $y = a(x-h)^2 + k$

(The easier form to use to find the vertex)

Vertex: (h, k)

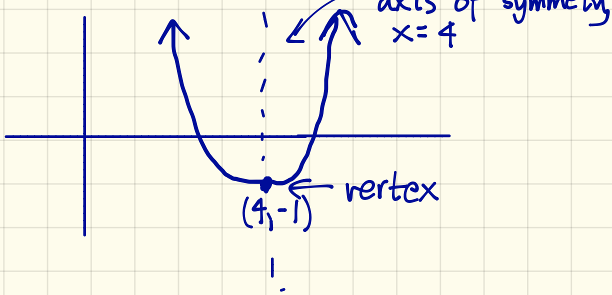
Axis of Symmetry: $x = h$.

[Examples] Identify the vertex & axis of symmetry and Sketch a graph.

① $y = \frac{1}{2}(x-4)^2 - 1$

vertex $= (4, -1)$

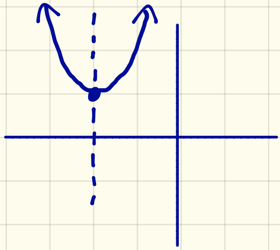
axis of symmetry $=$ $x = 4$



② $y=(x+2)^2+1$

Vertex: $(-2,1)$

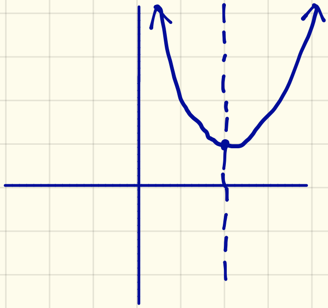
axis of symmetry: $x=-2$



③ $y=(x-2)^2+1$

vertex: $(2,1)$

axis of symmetry: $x=2$



Another way to find the vertex of a Quadratic.

Vertex Formula: $x=\frac{-b}{2a}$, $y=f\left(\frac{-b}{2a}\right)$.

[Examples] Find the vertex of the Quadratic.

① $y=x^2+8x+10$

$a=1$
$b=8$
$c=10$

$x=\frac{-b}{2a}=\frac{-8}{2(1)}=-4$

$y=f(-4)=(-4)^2+8(-4)+10=-6$

Vertex: $(-4,-6)$

② $y=x^2+10x+20$

$a=1$
$b=10$
$c=20$

$x=\frac{-b}{2a}=\frac{-10}{2(1)}=-5$

$y=f(-5)=(-5)^2+10(-5)+20$
$=-5$

Vertex: $(-5,-5)$

③ $y = -2x^2 - 16x - 32$

$a = -2$
$b = -16$
$c = -32$

$x = \frac{-b}{2a} = \frac{-(-16)}{2(-2)} = -4$

$y = f(-4) = -2(-4)^2 - 16(-4) - 32$
$= 0$

Vertex: $(-4, 0)$

④ $y = 3x^2 - 6x + 5$

$a = 3$
$b = -6$
$c = 5$

$x = \frac{-b}{2a} = \frac{-(-6)}{2(3)} = 1$

$y = f(1) = 3(1)^2 - 6(1) + 5$
$= 2$

Vertex: $(1, 2)$

TRANSFORMATION RULES FOR VERTEX FORM

$$y = a(x-h)^2 + k$$

Shift Up/Down

- $y = a(x-h) \underline{+k}$ (up)
- $y = a(x-h) \underline{-k}$ (down)

Shift Left/Right

- $y = a(x \underline{+h}) + k$ (left)
- $y = a(x \underline{-h}) + k$ (right)

SHRINKS/STRETCHES

- If $a > 1$, the graph stretches.
- If $0 < a < 1$, the graph shrinks.

Opens Up/Down

- If a is positive, opens upward
- If a is negative, opens downward
  (reflection across x-axis)

[Examples] Describe the transformation.

① $f(x)=(x-2)^2+3$
shift up 3, right 2,

② $f(x)=(x+5)^2-2$
shift left 5, down 2,

③ $f(x)=-x^2+10$
shift up 10,

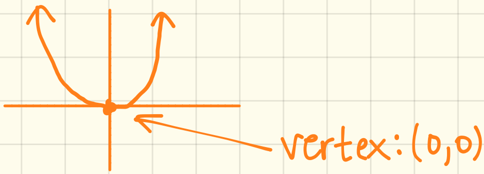
④ $f(x)=(x+5)^2$
shift left 5,

⑤ $f(x)=\frac{3}{4}(x-1)^2+5$
shift right 1, up 5, shrink by scale factor of $\frac{3}{4}$

⑥ $f(x)=-5(x+6)^2-7$
shift left 6, down 7, stretch by scale factor of 5,

Let's recall the parent function of a Quadratic Equation & its graph.

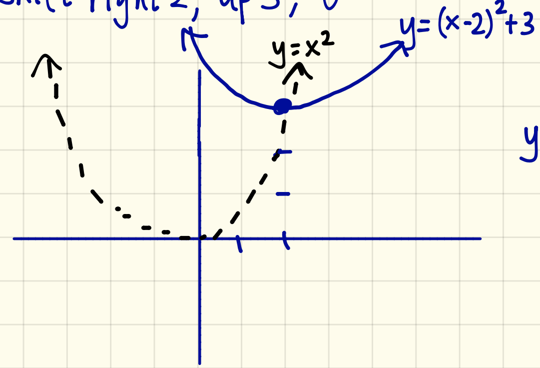
$f(x)=x^2$
(no transformations)



[Examples] Sketch the graph from the Quadratic equation.

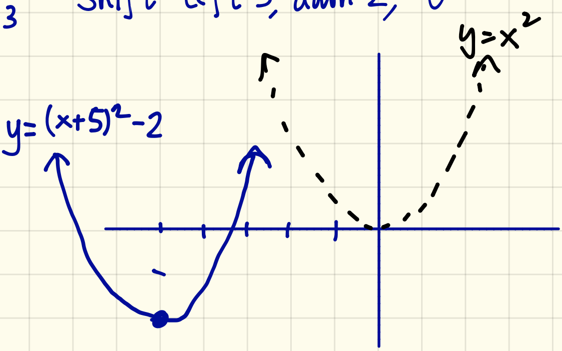
① $f(x)=(x-2)^2+3$

shift right 2, up 3,



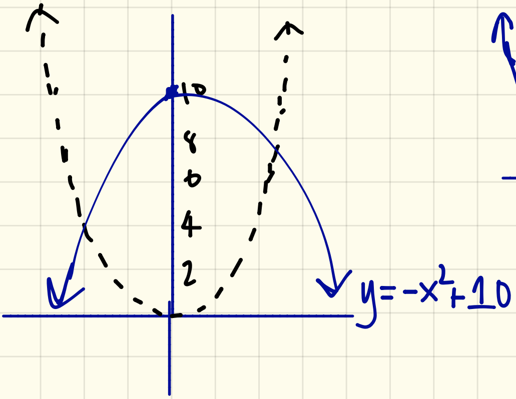
② $f(x)=(x+5)^2-2$

shift left 5, down 2,



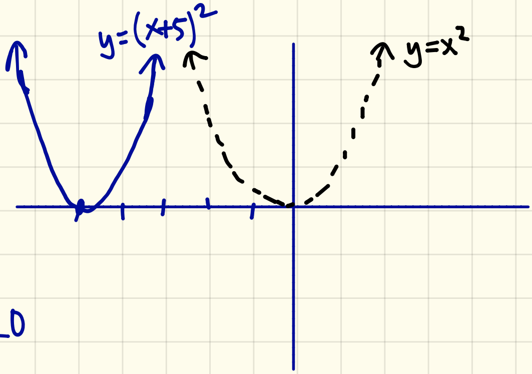
③ $f(x) = -x^2 + 10$

shift up 10,



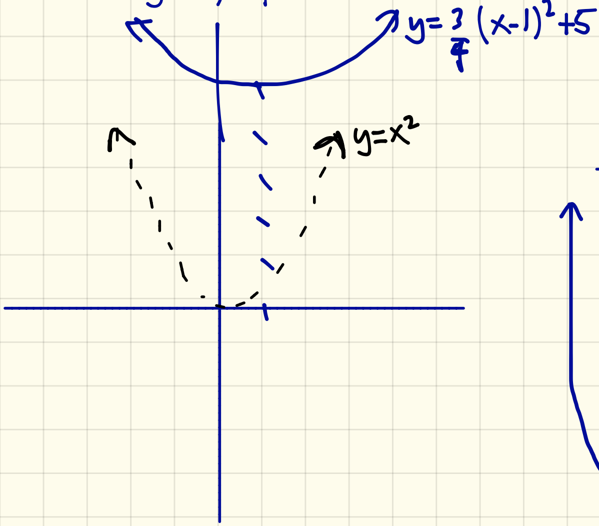
④ $f(x) = (x+5)^2$

shift left 5,



⑤ $f(x) = \frac{3}{4}(x-1)^2 + 5$

shift right 1, up 5, shrinks



⑥ $f(x) = -5(x+6)^2 - 7$

shift left 6, down 7, stretch